Worked Answers

Section One — Number

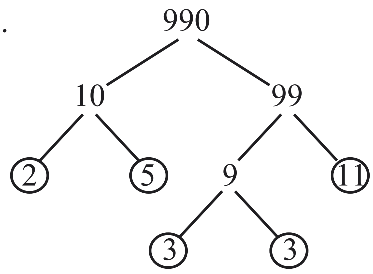
Page 2 — Types of Number and BODMAS

Q1 $3 + 22 \times 3 - 14 = 3 + 66 - 14 = 55$

[2 marks available — 1 mark for correct working, 1 mark for the correct answer.]

Page 3 — Multiples, Factors and Prime Factors

Q1 E.g.



$990 = 2 \times 3 \times 3 \times 5 \times 11 = 2 \times 3^2 \times 5 \times 11$

[2 marks available — 1 mark for correctly drawn factor tree (or other correct method), 1 mark for all prime factors correct.]

A correct method here is either using a factor tree or just repeatedly dividing the factors until you get primes.

Page 4 — LCM and HCF

Q1 **a)** Multiples of 9: 9, 18, 27, **36**, 45, 54, ...
Multiples of 12: 12, 24, **36**, 48, 60, ...
So the LCM of 9 and 12 is 36.
[2 marks available — 1 mark for correct method, 1 mark for correct answer.]

b) Prime factors that appear in either 28 or 8 are: 2, 2, 2, 7
LCM of 28 and 8 = $2 \times 2 \times 2 \times 7 = 56$
[2 marks available — 1 mark for correct method, 1 mark for correct answer.]

Q2 **a)** Find the factors of 36:
1×36
2×18
3×12
4×9
~~5 × =~~
6×6

6 is repeated, so stop.

Find the factors of 84:
1×84
2×42
3×28
4×21
~~5 × =~~
6×14
7×12
~~8 × =~~
~~9 × =~~
~~10 × =~~
~~11 × =~~
12×7

12 and 7 are repeated, so stop.
The factors of 36 are:
1, 2, 3, 4, 6, 9, **12**, 18, 36
The factors of 84 are:
1, 2, 3, 4, 6, 7, **12**, 14, 21, 28, 42, 84

So the HCF of 36 and 84 is 12.
[2 marks available — 1 mark for correct method, 1 mark for correct answer.]

b) $150 = 2 \times 3 \times 5 \times 5$ $\quad 60 = 2 \times 2 \times 3 \times 5$
2, 3 and 5 are prime factors of both 150 and 60 so the HCF = $2 \times 3 \times 5 = 30$.
[1 mark for correct answer.]

Page 6 — Fractions

Q1 **a)**
$$\frac{3}{8} \times 1\frac{5}{12} = \frac{3}{8} \times \frac{17}{12} = \frac{1}{8} \times \frac{17}{4} = \frac{1 \times 17}{8 \times 4} = \frac{17}{32}$$

[3 marks available — 1 mark for converting the mixed number to an improper fraction, 1 mark for multiplying numerators and denominators separately, 1 mark for correct answer in its simplest form.]

Worked Answers

b) $1\frac{7}{9} \div 2\frac{2}{3} = \frac{16}{9} \div \frac{8}{3}$

$= \frac{16}{9} \times \frac{3}{8}$

$= \frac{2}{3} \times \frac{1}{1} = \frac{2}{3}$

[3 marks available — 1 mark for converting the mixed numbers to improper fractions, 1 mark for turning the second fraction upside down and multiplying, 1 mark for correct answer in its simplest form.]

c) $4\frac{1}{9} + 2\frac{2}{27} = \frac{37}{9} + \frac{56}{27}$

$= \frac{111}{27} + \frac{56}{27}$

$= \frac{111 + 56}{27}$

$= \frac{167}{27} = 6\frac{5}{27}$

[3 marks available — 1 mark for converting the mixed numbers to improper fractions (or dealing with the integer parts separately), 1 mark for finding a common denominator, 1 mark for correct answer.]

d) $5\frac{2}{3} - 9\frac{1}{4} = \frac{17}{3} - \frac{37}{4}$

$= \frac{68}{12} - \frac{111}{12}$

$= \frac{68 - 111}{12}$

$= -\frac{43}{12} = -3\frac{7}{12}$

[3 marks available — 1 mark for converting the mixed numbers to improper fractions (or dealing with the integer parts separately), 1 mark for finding a common denominator, 1 mark for correct answer.]

Q2 Number of vegetarian sandwiches = $\frac{7}{15} \times 30$

$= (30 \div 15) \times 7 = 2 \times 7 = 14$ *[1 mark]*

Number of cheese sandwiches = $\frac{3}{7} \times 14$

$= (14 \div 7) \times 3 = 2 \times 3 = 6$ *[1 mark]*

Page 7 — Fractions, Decimals and Percentages

Q1 a) $0.4 = \frac{4}{10} = \frac{2}{5}$ *[1 mark]*

b) $0.02 = \frac{2}{100} = \frac{1}{50}$ *[1 mark]*

c) $0.77 = \frac{77}{100}$ *[1 mark]*

d) $0.555 = \frac{555}{1000} = \frac{111}{200}$ *[1 mark]*

e) $5.6 = \frac{56}{10} = \frac{28}{5}$ *[1 mark]*

Q2 a) $57\% = 0.57$, $\frac{5}{9} = 5 \div 9 = 0.555...$

So 57% is greater than $\frac{5}{9}$. *[1 mark]*

b) $\frac{6}{25} = \frac{24}{100} = 0.24$

So $\frac{6}{25}$ is greater than 0.2. *[1 mark]*

c) $\frac{7}{8} = 7 \div 8 = 0.875$, $90\% = 0.9$

So 90% is greater than $\frac{7}{8}$. *[1 mark]*

Page 9 — Fractions and Recurring Decimals

Q1 $0.\dot{1}2\dot{6} = \frac{126}{999}$ *[1 mark]* $= \frac{14}{111}$ *[1 mark]*

Q2 Let $r = 0.\dot{0}\dot{7}$

Then $100r = 7.\dot{0}\dot{7}$ *[1 mark]*

So $100r - r = 7.\dot{0}\dot{7} - 0.\dot{0}\dot{7}$

$\Rightarrow 99r = 7$

$\Rightarrow r = \frac{7}{99}$ *[1 mark]*

Q3 $\frac{5}{111} = \frac{45}{999}$ *[1 mark]* $= 0.\dot{0}4\dot{5}$ *[1 mark]*

Page 11 — Estimating

Q1 a) Rounding to the nearest integer:

$\frac{4.23 \times 11.8}{7.7} \approx \frac{4 \times 12}{8}$ *[1 mark]*

$= 6$ *[1 mark]*

OR

Rounding to 1 s.f.:

$\frac{4.23 \times 11.8}{7.7} \approx \frac{4 \times 10}{8}$ *[1 mark]*

$= 5$ *[1 mark]*

b) 136 lies between 121 ($= 11^2$) and 144 ($= 12^2$), so $\sqrt{136}$ will lie between 11 and 12 *[1 mark]*. 136 is closer to 144 than 121, so $\sqrt{136} \approx 11.7$ *[1 mark]*.

Answers in the range 11.6-11.8 would be acceptable here — the actual value is 11.66190...

Worked Answers

Q2 **a)** Round the values of π and r:
$\pi = 3.14159... \approx 3$ and $r = 9 \approx 10$ ***[1 mark]***.
Now put the numbers into the formula:
$V \approx \frac{4}{3} \times 3 \times 10^3 = 4 \times 1000 = 4000 \text{ cm}^3$
[1 mark]

b) Bigger ***[1 mark]***
You rounded π down to 3, but rounded 9 up to 10 and cubed it, so the overall answer is bigger.

Page 12 — Bounds

Q1 Upper bound for distance = 200 + 0.5 = 200.5 m
Lower bound for distance = 200 – 0.5 = 199.5 m
Upper bound for time = 32.2 + 0.05 = 32.25 s
Lower bound for time = 32.2 – 0.05 = 32.15 s
Upper bound for speed = 200.5 ÷ 32.15
= 6.23639... m/s
Lower bound for speed = 199.5 ÷ 32.25
= 6.18604... m/s
Both the upper and lower bounds for speed round to 6.2 m/s to 1 d.p. so her speed is 6.2 m/s to 1 decimal place.
[5 marks available — 1 mark for finding the upper and lower bounds for distance, 1 mark for finding the upper and lower bounds for time, 1 mark for finding the upper bound for speed, 1 mark for finding the lower bound for speed and 1 mark for rounding to find the correct answer.]
Be careful with the bounds for speed — the upper bound for speed uses the upper bound for distance and the lower bound for time.

Page 14 — Standard Form

Q1 0.854 million = 854 000 = 8.54 × 100 000
$= 8.54 \times 10^5$ ***[1 mark]***
$0.00018 = 1.8 \times 0.0001 = 1.8 \times 10^{-4}$ ***[1 mark]***

Q2 **a)** $(3.2 \times 10^7) \div (1.6 \times 10^{-4})$
$= (3.2 \div 1.6) \times (10^7 \div 10^{-4})$ ***[1 mark]***
$= 2 \times 10^{7-(-4)}$
$= 2 \times 10^{11}$ ***[1 mark]***

b) $(6.7 \times 10^{10}) + (5.8 \times 10^{11})$
$= (0.67 \times 10^{11}) + (5.8 \times 10^{11})$ ***[1 mark]***
$= (0.67 + 5.8) \times 10^{11}$
$= 6.47 \times 10^{11}$ ***[1 mark]***
Don't forget to make sure your answer's in standard form. In part b), you might have ended up with 64.7×10^{10}. But the question asks for the answer in standard form, so this wouldn't get you both marks.

Q3 $2^{25} \times 5^{27} = 2^{25} \times 5^{25} \times 5^2$
$= (2 \times 5)^{25} \times 25$ ***[1 mark]***
$= 10^{25} \times 25$ ***[1 mark]*** $= 10^{25} \times 2.5 \times 10$
$= 2.5 \times 10^{26}$ ***[1 mark]***

Page 15 — Revision Questions

Q1 **a)** Integers are whole numbers — either positive or negative, or zero.
b) Rational numbers are numbers that can be written as fractions.
c) Prime numbers are numbers which will only divide by themselves or 1 (excluding 1 itself, which is not prime).

Q2 **a)** 7 + 8 ÷ 2 = 7 + 4 = 11
b) 7 ÷ (5 + 9) = 7 ÷ 14 = 0.5
c) $(2 - 5 \times 3)^2 = (2 - 15)^2 = (-13)^2 = 169$

Q3 Multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, ...
Multiples of 16: 16, 32, 48, 64, ...
Multiples of 12: 12, 24, 36, 48, 60, ...
The LCM of 6, 16 and 12 is 48, so Noah needs to buy 48 of each item.
Number of packs of buns = 48 ÷ 6 = 8
Number of packs of cheese slices = 48 ÷ 16 = 3
Number of packs of hot dogs = 48 ÷ 12 = 4

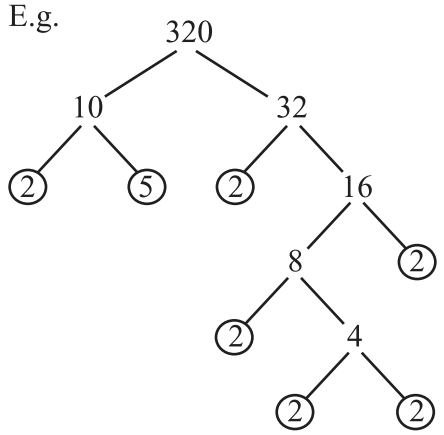
Q4 **a)** Find the factors of 28: 1 × 28
2 × 14
~~3 × =~~
4 × 7
~~5 × =~~
~~6 × =~~
7 × 4
7 and 4 are repeated, so stop.
Find the factors of 42: 1 × 42
2 × 21
3 × 14
~~4 × =~~
~~5 × =~~
6 × 7
7 × 6
7 and 6 are repeated, so stop.

The factors of 28 are: 1, 2, 4, 7, **14**, 28
The factors of 42 are: 1, 2, 3, 6, 7, **14**, 21, 42
So the HCF of 28 and 42 is 14.

b) Multiples of 8: 8, 16, 24, 32, **40**, 48, ...
Multiples of 10: 10, 20, 30, **40**, 50, 60, ...
So the LCM of 8 and 10 is 40.

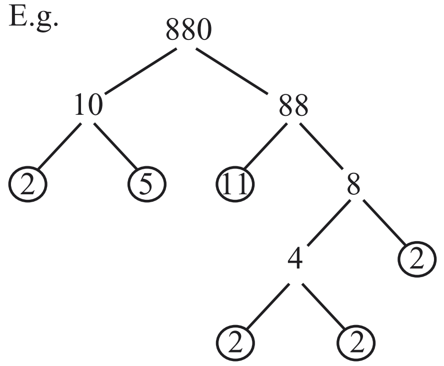
Worked Answers

Q5 **a)** E.g.



$320 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 = 2^6 \times 5$

E.g.



$880 = 2 \times 2 \times 2 \times 2 \times 5 \times 11 = 2^4 \times 5 \times 11$

b) LCM $= 2^6 \times 5 \times 11 = 3520$

HCF $= 2^4 \times 5 = 80$

Q6 Divide top and bottom by the same number till they won't go any further.

Q7 **a)** $74 \div 9 = 8$ remainder 2, so $\frac{74}{9} = 8\frac{2}{9}$

b) $4\frac{5}{7} = 4 + \frac{5}{7} = \frac{28}{7} + \frac{5}{7} = \frac{28+5}{7} = \frac{33}{7}$

Q8 Multiplying: Multiply top and bottom numbers separately.

Dividing: Turn the second fraction upside down, then multiply.

Adding/subtracting: Put fractions over a common denominator, then add/subtract the numerators.

Q9 **a)** $\frac{2}{11} \times \frac{7}{9} = \frac{2 \times 7}{11 \times 9} = \frac{14}{99}$

b) $5\frac{1}{2} \div 1\frac{3}{4} = \frac{11}{2} \div \frac{7}{4}$

$= \frac{11}{2} \times \frac{4}{7}$

$= 11 \times \frac{2}{7}$

$= \frac{22}{7}$ or $3\frac{1}{7}$

c) $\frac{5}{8} - \frac{1}{6} = \frac{15}{24} - \frac{4}{24}$

$= \frac{15-4}{24}$

$= \frac{11}{24}$

d) $3\frac{3}{10} + 4\frac{1}{4} = \frac{33}{10} + \frac{17}{4}$

$= \frac{66}{20} + \frac{85}{20}$

$= \frac{151}{20}$ or $7\frac{11}{20}$

Q10 **a)** $\frac{7}{9}$ of 270 kg $= (270 \div 9) \times 7 = 30 \times 7$
$= 210$ kg

b) $\frac{88}{56} = \frac{11}{7}$

Q11 $\frac{3}{4} = \frac{30}{40}$, $\frac{5}{8} = \frac{25}{40}$ and $\frac{7}{10} = \frac{28}{40}$,

so $\frac{7}{10}$ is closer to $\frac{3}{4}$.

Q12 **a)** Divide the numerator by the denominator.

b) Put the digits after the decimal point on the top, and a power of 10 with the same number of zeros as there were decimal places on the bottom.

Q13 **a)** **(i)** $\frac{4}{100} = \frac{1}{25}$ **(ii)** 4%

b) **(i)** $\frac{65}{100} = \frac{13}{20}$ **(ii)** 0.65

Q14 Orange juice = 50% of 25 = 12.5 litres

Lemonade = $\frac{2}{5}$ of 25 = 10 litres

Cranberry juice = $\frac{1}{10}$ of 25 = 2.5 litres

Q15 Let $r = 0.\dot{5}\dot{1}$

Then $100r = 51.\dot{5}\dot{1}$

So $100r - r = 51.\dot{5}\dot{1} - 0.\dot{5}\dot{1}$

$\Rightarrow 99r = 51$

$\Rightarrow r = \frac{51}{99} = \frac{17}{33}$

Q16 **a)** The decider is 3, so leave the last digit as 6, giving 427.96

b) The decider is 6, so round up to 428.0

c) The decider is 7, so round up to 430

d) The decider is 6, so round up to 428.0

Q17 E.g. $(104.6 + 56.8) \div 8.4$

$\approx (100 + 60) \div 8$

$= 160 \div 8$

$= 20$

Depending on how you've rounded, you might have a different estimate — your answer should be in the range 16-20.

Worked Answers

Q18 45 lies between 36 (= 6^2) and 49 (= 7^2), so $\sqrt{45}$ will lie between 6 and 7. 45 is closer to 49 than 36, so $\sqrt{45} \approx 6.7$.
Estimates between 6.6 and 6.8 are OK here — the actual value is 6.70820...

Q19 The upper and lower bounds of a rounded measurement are half a unit either side of the rounded value. The upper and lower bounds of a truncated measurement are the truncated value itself and a whole unit above the truncated value.

Q20 Upper bound = 2.4 litres + 0.05 litres
= 2.45 litres
Lower bound = 2.4 litres – 0.05 litres
= 2.35 litres
As an inequality (with volume = V), this is
2.35 litres ≤ V < 2.45 litres

Q21 Upper bound of length = 15.6 + 0.05 = 15.65 m
Upper bound of width = 8.4 + 0.05 = 8.45 m
So maximum area = 15.65 × 8.45 = 132.2425 m^2

Q22 1. The front number must always be between 1 and 10.
2. The power of 10, n, is how far the decimal point moves.
3. n is positive for big numbers, and negative for small numbers.

Q23 **a)** $970\,000 = 9.7 \times 100\,000 = 9.7 \times 10^5$
b) $3\,560\,000\,000 = 3.56 \times 1\,000\,000\,000$
$= 3.56 \times 10^9$
c) $0.00000275 = 2.75 \times 0.000001$
$= 2.75 \times 10^{-6}$

Q24 $4.56 \times 10^{-3} = 4.56 \times 0.001 = 0.00456$
$2.7 \times 10^5 = 2.7 \times 100\,000 = 270\,000$

Q25 **a)** $(3.2 \times 10^6) \div (1.6 \times 10^3)$
$= (3.2 \div 1.6) \times (10^6 \div 10^3)$
$= 2 \times 10^{6-3}$
$= 2 \times 10^3$
b) $(1.75 \times 10^{12}) + (9.89 \times 10^{11})$
$= (1.75 \times 10^{12}) + (0.989 \times 10^{12})$
$= (1.75 + 0.989) \times 10^{12}$
$= 2.739 \times 10^{12}$

Q26 After 30 minutes: $(3.1 \times 10^8) \times 2^3 = 24.8 \times 10^8$
$= 2.48 \times 10^9$

Section Two — Algebra

Page 16 — Algebra Basics

Q1 Perimeter = $5x + (3y + 1) + 5x + (3y + 1)$
$= 10x + 6y + 2$ cm
[2 marks available — 1 mark for a correct unsimplified expression, 1 mark for correct simplified answer.]

Page 17 — Powers and Roots

Q1 **a)** $e^4 \times e^7 = e^{4+7} = e^{11}$ ***[1 mark]***
b) $f^9 \div f^5 = f^{9-5} = f^4$ ***[1 mark]***
c) $(g^6)^{\frac{1}{2}} = g^{6 \times \frac{1}{2}} = g^3$ ***[1 mark]***
d) $2h^5j^{-2} \times 3h^2j^4 = (2 \times 3)h^{5+2}j^{-2+4} = 6h^7j^2$
[2 marks available — 2 marks for the correct answer, otherwise 1 mark for 2 of 6, h^7 and j^2 correct.]

Q2 **a)** $625^{\frac{3}{4}} = (625^{\frac{1}{4}})^3 = (\sqrt[4]{625})^3 = 5^3 = 125$
[2 marks available — 1 mark for finding the 4th root, 1 mark for correct answer.]
b) $25^{-\frac{1}{2}} = \frac{1}{25^{\frac{1}{2}}} = \frac{1}{\sqrt{25}} = \frac{1}{5}$
[2 marks available — 1 mark for use of reciprocal or square root, 1 mark for correct answer.]
c) $\left(\frac{27}{216}\right)^{-\frac{1}{3}} = \left(\frac{216}{27}\right)^{\frac{1}{3}} = \frac{\sqrt[3]{216}}{\sqrt[3]{27}} = \frac{6}{3} = 2$
[2 marks available — 1 mark for use of reciprocal or cube root, 1 mark for correct answer.]
You might have to use some trial and error to find the roots if you didn't know that $6^3 = 216$ and $5^4 = 625$.

Page 18 — Multiplying Out Brackets

Q1 **a)** $(y + 4)(y - 5) = y^2 - 5y + 4y - 20$
$= y^2 - y - 20$
[2 marks available — 1 mark for first step with or without correct signs, 1 mark for all terms correct.]
b) $(2p - 3)^2 = (2p - 3)(2p - 3)$
$= 4p^2 - 6p - 6p + 9$
$= 4p^2 - 12p + 9$
[2 marks available — 1 mark for brackets multiplied out, with or without correct signs, 1 mark for all terms correct.]

Q2 **a)** $(2t + \sqrt{2})(t - 3\sqrt{2})$
$= 2t^2 - 6t\sqrt{2} + t\sqrt{2} - 6$
$= 2t^2 - 5t\sqrt{2} - 6$
[3 marks available — 1 mark for each correct term.]
$\sqrt{2} \times 3\sqrt{2} = 3(\sqrt{2})^2 = 3 \times 2 = 6$

Worked Answers

b) $(x-2)^3 = (x-2)(x-2)(x-2)$
$= (x-2)(x^2-4x+4)$
$= x^3 - 4x^2 + 4x - 2x^2 + 8x - 8$
$= x^3 - 6x^2 + 12x - 8$
[3 marks available — 1 mark for multiplying two sets of brackets correctly, 1 mark for multiplying product by the third bracket, 1 mark for correct answer.]

Page 19 — Factorising

Q1 $3y(2x + 5y)$
[2 marks available — 1 mark for each correct factor.]
Remember, it's easy to check that you've factorised correctly — just multiply out the brackets and make sure you get back to the original expression.

Q2 $x^2 - 16y^2 = (x + 4y)(x - 4y)$
[2 marks available — 1 mark for using the difference of two squares, 1 mark for correct answer.]

Q3 $x^2 - 11 = (x + \sqrt{11})(x - \sqrt{11})$
[2 marks available — 1 mark for using the difference of two squares, 1 mark for correct answer.]

Q4 $\dfrac{6x-42}{x^2-49} = \dfrac{6(x-7)}{(x+7)(x-7)} = \dfrac{6}{x+7}$
[3 marks available — 1 mark for factorising numerator, 1 mark for factorising denominator, 1 mark for correct answer.]

Page 20 — Manipulating Surds

Q1 $\sqrt{180} = \sqrt{36 \times 5} = \sqrt{36}\sqrt{5} = 6\sqrt{5}$
$\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4}\sqrt{5} = 2\sqrt{5}$
$(\sqrt{5})^3 = \sqrt{5} \times \sqrt{5} \times \sqrt{5} = 5\sqrt{5}$
So
$\sqrt{180} + \sqrt{20} + (\sqrt{5})^3 = 6\sqrt{5} + 2\sqrt{5} + 5\sqrt{5}$
$= 13\sqrt{5}$
[3 marks available — 2 marks for correctly simplifying all three surds (or 1 mark for one or two surds simplified correctly), 1 mark for the correct answer.]

Q2 $\dfrac{2}{2+\sqrt{3}} = \dfrac{2(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})} = \dfrac{4-2\sqrt{3}}{4-3}$
$= 4 - 2\sqrt{3}$
[3 marks available — 1 mark for attempt at rationalising the denominator, 1 mark for correct value of a, 1 mark for correct value of b.]

Page 21 — Solving Equations

Q1 $2x + 5 = 17 - 4x$
$2x = 12 - 4x$
$6x = 12$
$x = 2$
[2 marks available — 1 mark for rearranging equation, 1 mark for correct answer.]

Q2 $4(y + 3) = 3y + 16$
$4y + 12 = 3y + 16$
$y = 4$
[3 marks available — 1 mark for multiplying out bracket, 1 mark for rearranging equation, 1 mark for correct answer.]

Q3 $\dfrac{3x+2}{5} = \dfrac{5x+6}{9}$
$9(3x + 2) = 5(5x + 6)$
$27x + 18 = 25x + 30$
$2x = 12$
$x = 6$
[3 marks available — 1 mark for getting rid of fractions, 1 mark for multiplying out brackets, 1 mark for correct answer.]

Page 22 — Solving Equations

Q1 $2x^2 + 8 = 80$
$2x^2 = 72$
$x^2 = 36$
$x = \pm 6$
[2 marks available — 1 mark for rearranging equation, 1 mark for correct answer.]

Q2 $\dfrac{3x-2}{2} - \dfrac{4x-5}{3} = 2$
$3(3x - 2) - 2(4x - 5) = 12$
$9x - 6 - 8x + 10 = 12$
$x = 8$
[3 marks available — 1 mark for getting rid of fractions, 1 mark for multiplying out brackets, 1 mark for correct answer.]

Page 23 — Rearranging Formulas

Q1 $p = \dfrac{q}{7} + 2r$
$7p = q + 14r$
$q = 7(p - 2r)$ or $7p - 14r$
[2 marks available — 1 mark for getting rid of fraction, 1 mark for correct answer.]

Worked Answers

Q2 $a = \frac{v-u}{t}$

$at = v - u$

$v = u + at$

[2 marks available — 1 mark for getting rid of fraction, 1 mark for correct answer.]

Page 24 — Rearranging Formulas

Q1 a) $x = \frac{y^2}{4}$

$4x = y^2$

$y = \pm 2\sqrt{x}$

[2 marks available — 2 marks for fully correct answer, otherwise 1 mark for partially correct answer, e.g. $2\sqrt{x}$ or $\pm\sqrt{4x}$.]

Remember the ± when you take a square root.

b) $x = \frac{y}{y-z}$

$x(y - z) = y$

$xy - xz = y$

$xy - y = xz$

$y(x - 1) = xz$

$y = \frac{xz}{x-1}$

[4 marks available — 1 mark for getting rid of fraction, 1 mark for rearranging equation, 1 mark for factorising for y, 1 mark for correct answer.]

Page 25 — Factorising Quadratics

Q1 $(x + 5)(x - 3)$

[2 marks available — 1 mark for including 5 and 3 in the brackets, 1 mark for correct answer.]

Q2 $x^2 - 9x + 20 = 0$

$(x - 4)(x - 5) = 0$

$x = 4$ or $x = 5$

[3 marks available — 1 mark for including 4 and 5 in the brackets, 1 mark for correct signs in the brackets, 1 mark for correct solutions to the equation.]

Always check your factorisation by multiplying out the brackets.

Page 26 — Factorising Quadratics

Q1 $2x^2 - 5x - 12 = (2x + 3)(x - 4)$

[2 marks available — 1 mark for $(2x \pm 3)(x \pm 4)$, 1 mark for correct answer.]

Q2 $3x^2 + 10x - 8 = 0$

$(3x - 2)(x + 4) = 0$

$x = \frac{2}{3}$ or $x = -4$

[3 marks available — 1 mark for $(3x \pm 2)(x \pm 4)$, 1 mark for both factors correct, 1 mark for correct answer.]

Q3 $3x^2 + 32x + 20 = (3x + 2)(x + 10)$

[2 marks available — 1 mark for $(3x \pm 2)(x \pm 10)$, 1 mark for correct answer.]

Q4 $5x^2 - 13x = 6$

$5x^2 - 13x - 6 = 0$

$(5x + 2)(x - 3) = 0$

$x = -\frac{2}{5}$ or $x = 3$

[3 marks available — 1 mark for $(5x \pm 2)(x \pm 3)$, 1 mark for both factors correct, 1 mark for correct answer.]

Page 27 — The Quadratic Formula

Q1 $x^2 + 10x - 4 = 0$

$a = 1, b = 10, c = -4$

$$x = \frac{-10 \pm \sqrt{10^2 - 4 \times 1 \times (-4)}}{2 \times 1}$$

$$x = \frac{-10 \pm \sqrt{116}}{2}$$

$x = 0.39$ or -10.39 (2 d.p.)

[3 marks available — 1 mark for correct substitution into quadratic formula, 1 mark for simplification, 1 mark for correct answer.]

Q2 $2x + \frac{3}{x-2} = -2$

$2x(x - 2) + 3 = -2(x - 2)$

$2x^2 - 4x + 3 = -2x + 4$

$2x^2 - 2x - 1 = 0$

Use quadratic formula: $a = 2, b = -2, c = -1$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 2 \times (-1)}}{2 \times 2}$$

$$x = \frac{2 \pm \sqrt{12}}{4} = \frac{2 \pm 2\sqrt{3}}{4}$$

$$x = \frac{1 \pm \sqrt{3}}{2}$$

[4 marks available — 1 mark for multiplying by $(x - 2)$, 1 mark for multiplying out and rearranging into standard quadratic form, 1 mark for correct substitution into quadratic formula, 1 mark for correct answer.]

Worked Answers

Page 28 — Completing the Square

Q1 $(x-6)^2 = x^2 - 12x + 36$
$= x^2 - 12x + 23 + 13$
So $x^2 - 12x + 23 = (x-6)^2 - 13$
[3 marks available — 3 marks for the correct answer, otherwise 1 mark for correct squared bracket, 1 mark for correct adjusting number.]

Q2 $x^2 + 10x + 7 = 0$
$(x+5)^2 - 25 + 7 = 0$
$(x+5)^2 - 18 = 0$
$(x+5)^2 = 18$
$(x+5) = \pm\sqrt{18}$
$x = -5 \pm \sqrt{18}$
$x = -5 \pm 3\sqrt{2}$
[5 marks available — 1 mark for writing $(x+5)^2$, 1 mark for completing the square correctly, 1 mark for rearranging to find x, 1 mark for simplifying the surd, 1 mark for correct answer.]
Make sure you remember the ± when you take the square root.

Page 29 — Completing the Square

Q1 **a)** $2x^2 + 3x - 5 = 2(x^2 + \frac{3}{2}x) - 5$
$= 2(x + \frac{3}{4})^2 - \frac{9}{8} - 5$
$= 2(x + \frac{3}{4})^2 - \frac{49}{8}$
[4 marks available — 1 mark for taking 2 outside a bracket, 1 mark each for the two correct terms in the bracket, 1 mark for correct adjusting number.]

b) $2x^2 + 3x - 5 = 0$
$2(x + \frac{3}{4})^2 - \frac{49}{8} = 0$
$2(x + \frac{3}{4})^2 = \frac{49}{8}$
$(x + \frac{3}{4})^2 = \frac{49}{16}$
$x + \frac{3}{4} = \pm\sqrt{\frac{49}{16}} = \pm\frac{7}{4}$
$x = -\frac{3}{4} \pm \frac{7}{4}$
$x = 1, x = -\frac{5}{2}$
[2 marks available — 1 mark for a correct method, 1 mark for both values of x.]

c) Minimum point occurs when the bit in the brackets = 0, i.e. when $x = -\frac{3}{4}$.
So the coordinates of the minimum point are $(-\frac{3}{4}, -\frac{49}{8})$ ***[1 mark]***.

Page 30 — Algebraic Fractions

Q1 $$\frac{x^4 - 4y^2}{x^3 - 2xy} = \frac{(x^2+2y)(x^2-2y)}{x(x^2-2y)} = \frac{x^2+2y}{x}$$
[3 marks available — 1 mark for correct factorisation of numerator, 1 mark for correct factorisation of denominator, 1 mark for correct answer.]

Q2 $$\frac{x^2-3x-10}{x^3-2x^2} \div \frac{x^2-25}{6x-12}$$
$$= \frac{x^2-3x-10}{x^3-2x^2} \times \frac{6x-12}{x^2-25}$$
$$= \frac{(x-5)(x+2)}{x^2(x-2)} \times \frac{6(x-2)}{(x+5)(x-5)}$$
$$= \frac{x+2}{x^2} \times \frac{6}{x+5} = \frac{6(x+2)}{x^2(x+5)}$$
[6 marks available — 1 mark for turning the second fraction upside down and multiplying, 1 mark for factorising numerator of first fraction, 1 mark for factorising denominator of first fraction, 1 mark for fully factorising second fraction, 1 mark for cancelling common factors, 1 mark for correct answer.]

Q3 $$\frac{2}{x+5} + \frac{3}{x-2} = \frac{2(x-2)+3(x+5)}{(x-2)(x+5)}$$
$$= \frac{2x-4+3x+15}{(x-2)(x+5)}$$
$$= \frac{5x+11}{(x-2)(x+5)}$$
[3 marks available — 1 mark for putting over common denominator, 1 mark for simplifying numerator, 1 mark for correct answer.]

Page 31 — Sequences

Q1 n: 1 2 3 4
term: 2 9 16 23
Difference: 7 7 7 ***[1 mark]***
7n: 7, 14, 21, 28
To get from 7n to the term, you have to subtract 5. So nth term = 7n – 5 ***[1 mark]***

Q2 Sequence: 6 10 18 30
First difference: 4 8 12
Second difference: 4 4
Coefficient of $n^2 = 4 \div 2 = 2$.
Sequence given by $2n^2$: 2 8 18 32
Actual sequence – $2n^2$ sequence:
4 2 0 –2
Difference: –2 –2 –2
So this is a linear sequence with nth term –2n + 6.
So the nth term of the original sequence is $2n^2 - 2n + 6$.

Worked Answers

[4 marks available — 1 mark for finding the second differences, 1 mark for finding the difference between the actual sequence and the $2n^2$ sequence, 1 mark for finding the linear sequence of the differences and 1 mark for the correct answer.]

Page 32 — Sequences

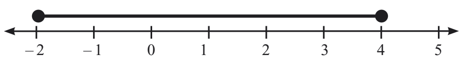
Q1 Let the first term be a.
a + (a + 8) + (a + 16) = 126
3a + 24 = 126 ***[1 mark]***
3a = 102
a = 34 ***[1 mark]***
So the three terms are 34, 34 + 8 = 42 ***[1 mark]*** and 42 + 8 = 50 ***[1 mark]***.

Page 33 — Inequalities

Q1 **a)** $11x + 3 < 42 - 2x$
$11x < 39 - 2x$
$13x < 39$
$x < 3$
[2 marks available — 1 mark for rearranging the inequality, 1 mark for correct answer.]

b) $6 - 4x \geq 18$
$-4x \geq 12$
$x \leq -3$
[2 marks available — 1 mark for rearranging the inequality, 1 mark for correct answer.]

Q2 $-8 \leq 5x + 2 \leq 22$
$-10 \leq 5x \leq 20$
$-2 \leq x \leq 4$



[3 marks available — 1 mark for rearranging the inequality, 1 mark for correct answer, 1 mark for number line.]

Page 34 — Inequalities

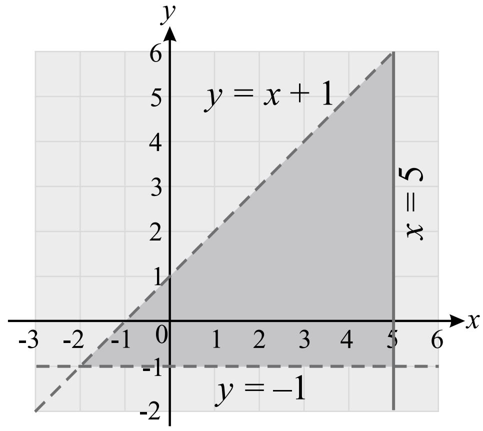
Q1 **a)** If $p^2 = 49$, then $p = 7$ or $p = -7$ ***[1 mark]***.
As $p^2 < 49$, $-7 < p < 7$ ***[1 mark]***.

b) $-\frac{1}{2}p^2 \leq -32$ means that $p^2 \geq 64$ ***[1 mark]***.
If $p^2 = 64$, then $p = 8$ or $p = -8$ ***[1 mark]***.
As $p^2 \geq 64$, $p \leq -8$ or $p \geq 8$ ***[1 mark]***.

Q2 $x^2 - 4x = 0$ factorises to give $x(x - 4) = 0$.
The graph of $y = x^2 - 4x$ is a u-shaped curve that crosses the x-axis at $x = 0$ and $x = 4$, and the curve is below the x-axis between these points, so $0 \leq x \leq 4$. The integer values that satisfy this inequality are $x = 0, 1, 2, 3$ and 4.
[3 marks available — 1 mark for factorising the quadratic, 1 mark for finding where it's below the x-axis and 1 mark for the correct solution.]

Page 35 — Graphical Inequalities

Q1



[3 marks available — 1 mark for 2 lines correct, 1 mark for all lines correct, 1 mark for correct shaded region.]

To make sure you've shaded the correct region, always double-check which side of each line satisfies the inequality.

Page 36 — Iterative Methods

Q1 Using $x_0 = 2$
$x_1 = 1.91293...$
$x_2 = 1.88883...$
$x_3 = 1.88205...$
$x_4 = 1.88014...$
x_3 and x_4 both round to 1.88 to 2 d.p. so $x = 1.88$ to 2 d.p.
[4 marks available — 1 mark for finding x_1 and x_2, 1 mark for finding x_3 and x_4, 1 mark for finding two terms that are the same to 2 d.p., 1 mark for correct answer.]

Worked Answers

Page 37 — Simultaneous Equations 1

Q1 Let t = cost of one cup of tea and c = cost of one slice of cake

$2t + 3c = 9$ (1)
$4t + c = 8$ (2)

(1) × 2: $4t + 6c = 18$ (3)
(3) – (2): $5c = 10$
$c = 2$
Sub into (1): $2t + 3 \times 2 = 9$
$2t = 3$, so $t = 1.5$

So a cup of tea costs £1.50 and a slice of cake costs £2.

[3 marks available — 1 mark for eliminating one of the variables, 1 mark for correct value of c, 1 mark for correct value of t.]

An equally correct method would be to first eliminate c in order to find t and then to substitute the value of t back into one of the equations to find c.

Q2 $2x - 10 = 4y \rightarrow 2x - 4y = 10$ (1)
$3y = 5x - 18 \rightarrow 5x - 3y = 18$ (2)

(1) × 5: $10x - 20y = 50$ (3)
(2) × 2: $10x - 6y = 36$ (4)

(3) – (4): $-14y = 14$
$y = -1$
Sub into (1): $2x - 4 \times (-1) = 10$
$2x = 6$
$x = 3$

Solution: $x = 3$, $y = -1$

[3 marks available — 1 mark for eliminating one of the variables, 1 mark for correct value of x, 1 mark for correct value of y.]

An equally correct method would be to first eliminate y in order to find x and then to substitute the value of x back into one of the equations to find y.

Page 38 — Simultaneous Equations 2

Q1 $y = 2 - 3x$ (1)
$y + 2 = x^2 \rightarrow y = x^2 - 2$ (2)

Sub (2) into (1): $x^2 - 2 = 2 - 3x$ (3)
$x^2 + 3x - 4 = 0$
$(x - 1)(x + 4) = 0$
$x = 1$ or $x = -4$
Sub $x = 1$ into (1): $y = 2 - 3 \times 1 = -1$
Sub $x = -4$ into (1): $y = 2 - 3 \times (-4) = 14$

Solutions: $x = 1$, $y = -1$ and $x = -4$, $y = 14$

[4 marks available — 1 mark for substituting to get a quadratic equation, 1 mark for factorising, 1 mark for each pair of values for x and y.]

It's a good idea to check your solutions by substituting them into the equation you didn't use when you found the second variable — here it'd be equation (2).

Q2 $y = x^2 + 4$ (1)
$y - 6x - 4 = 0$ (2)

Sub (1) into (2): $x^2 + 4 - 6x - 4 = 0$ (3)
$x^2 - 6x = 0$
$x(x - 6) = 0$
$x = 0$ or $x = 6$
Sub $x = 0$ into (2): $y - (6 \times 0) - 4 = 0$
$y = 4$
Sub $x = 6$ into (2): $y - (6 \times 6) - 4 = 0$
$y = 40$

So the coordinates of the points of intersection are A (0, 4) and B (6, 40).
To find the length of the line AB, use Pythagoras' theorem:
$\text{length}^2 = (6 - 0)^2 + (40 - 4)^2$
$= 6^2 + 36^2 = 1332$

So length $= \sqrt{1332}$ units

The question asked for an exact length, so leave your answer as a square root.

[5 marks available — 1 mark for substituting to get a quadratic equation, 1 mark for factorising, 1 mark for each correct pair of coordinates, 1 mark for correct length of AB.]

Page 39 — Proof

Q1 Take two consecutive even numbers, $2n$ and $2n + 2$, where n is an integer.
Then $2n + (2n + 2) = 4n + 2 = 2(2n + 1)$, which is even as $(2n + 1)$ is an integer.

[3 marks available — 1 mark for writing even number as 2n, 1 mark for adding 2n and 2n + 2, 1 mark for showing result is even.]

Sums and products of integers are always integers, so if n is an integer, so is 2n + 1.

Q2 $4x + 2 = 3(3a + x)$, so $x = 9a - 2$. If a is odd, then $9a$ is also odd (as odd × odd = odd). $9a - 2$ is always odd (as odd – even = odd), so x cannot be a multiple of 8 as all multiples of 8 are even.

[3 marks available — 1 mark for rearranging to find an expression for x, 1 mark for showing that x is always odd, 1 mark for stating that multiples of 8 are even so x cannot be a multiple of 8.]

Worked Answers

Page 40 — Proof

Q1 Take two consecutive integers, n and $n + 1$ and square them to get n^2 and $n^2 + 2n + 1$. The difference between them is $2n + 1$, which is an odd integer.
[3 marks available — 1 mark for squaring two consecutive numbers correctly, 1 mark for working out difference, 1 mark for showing difference is odd.]

Q2 Take the nth and $(n + 1)$th triangle numbers, $\frac{1}{2}n(n + 1)$ and $\frac{1}{2}(n + 1)(n + 2)$. Their ratio is $\frac{1}{2}n(n + 1) : \frac{1}{2}(n + 1)(n + 2)$, which simplifies to $n : n + 2$.
[2 marks available — 1 mark for a correct expression for the (n+1)th triangle number, 1 mark for simplifying ratio.]

Page 41 — Functions

Q1 **a)** $f(4) = 5(4) - 1 = 20 - 1 = 19$ ***[1 mark]***
b) $h(-2) = (-2)^2 + 3 = 4 + 3 = 7$ ***[1 mark]***
c) $gf(x) = g(f(x)) = g(5x - 1)$ ***[1 mark]***
$= 8 - 2(5x - 1) = 8 - 10x + 2$
$= 10 - 10x$ ***[1 mark]***
d) $fh(x) = f(h(x)) = f(x^2 + 3)$ ***[1 mark]***
$= 5(x^2 + 3) - 1 = 5x^2 + 15 - 1$
$= 5x^2 + 14$ ***[1 mark]***
e) $gh(-3) = g(h(-3)) = g((-3)^2 + 3)$
$= g(12)$ ***[1 mark]***
$= 8 - 2(12) = -16$ ***[1 mark]***
f) $x = f(y)$:
$x = 5y - 1$ ***[1 mark]***
$x + 1 = 5y$
$y = \frac{x+1}{5}$ ***[1 mark]***
Then replace y with $f^{-1}(x)$:
$f^{-1}(x) = \frac{x+1}{5}$ ***[1 mark]***

Page 42 — Revision Questions

Q1 $3x + 2y - 5 - 6y + 2x = 5x - 4y - 5$

Q2 **a)** $x^3 \times x^6 = x^{3+6} = x^9$
b) $y^7 \div y^5 = y^{7-5} = y^2$
c) $(z^3)^4 = z^{3 \times 4} = z^{12}$

Q3 **a)** $3(2x + 1) = (3 \times 2x) + (3 \times 1) = 6x + 3$
b) $(x + 2)(x - 3) = x^2 - 3x + 2x - 6 = x^2 - x - 6$
c) $(x - 1)(x + 3)(x + 5) = (x - 1)(x^2 + 8x + 15)$
$= x^3 + 8x^2 + 15x - x^2 - 8x - 15$
$= x^3 + 7x^2 + 7x - 15$
You could have multiplied the first two brackets together first instead — you'd end up with the same answer.

Q4 **a)** $8x^2 - 2y^2 = 2(4x^2 - y^2) = 2(2x + y)(2x - y)$
b) $(7 + 9pq)(7 - 9pq)$
c) $12x^2 - 48y^2 = 12(x^2 - 4y^2) = 12(x + 2y)(x - 2y)$

Q5 **a)** $\sqrt{27} = \sqrt{9 \times 3} = \sqrt{9} \times \sqrt{3} = 3\sqrt{3}$
b) $\frac{\sqrt{125}}{\sqrt{5}} = \sqrt{\frac{125}{5}} = \sqrt{25} = 5$

Q6 $\sqrt{98} + 3\sqrt{8} - \sqrt{200}$
$= \sqrt{49 \times 2} + 3\sqrt{4 \times 2} - \sqrt{100 \times 2}$
$= \sqrt{49}\sqrt{2} + 3\sqrt{4}\sqrt{2} - \sqrt{100}\sqrt{2}$
$= 7\sqrt{2} + 6\sqrt{2} - 10\sqrt{2} = 3\sqrt{2}$

Q7 **a)** $5(x + 2) = 8 + 4(5 - x)$
$5x + 10 = 8 + 20 - 4x$
$9x = 18$
$x = 2$
b) $x^2 - 21 = 3(5 - x^2)$
$x^2 - 21 = 15 - 3x^2$
$4x^2 = 36$
$x^2 = 9$
$x = \pm 3$

Q8 **a)** $\frac{p}{p+y} = 4$
$p = 4(p + y)$
$p = 4p + 4y$
$-3p = 4y$
$p = -\frac{4y}{3}$
b) $\frac{1}{p} = \frac{1}{q} + \frac{1}{r}$
$qr = pr + pq$
$qr = p(r + q)$
$p = \frac{qr}{r+q}$

Q9 **a)** $x^2 + 9x + 18 = 0$
$(x + 3)(x + 6) = 0$
$x = -3$ or $x = -6$
b) $5x^2 - 17x - 12 = 0$
$(5x + 3)(x - 4) = 0$
$5x = -3$ or $x = 4$
$x = -\frac{3}{5}$ or $x = 4$

Q10 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Worked Answers

Q11 a) $x^2 + x - 4 = 0$

a = 1, b = 1, c = –4

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times (-4)}}{2 \times 1}$$

$$x = \frac{-1 \pm \sqrt{17}}{2}$$

$x = 1.56$ or $x = -2.56$ (2 d.p.)

b) $5x^2 + 6x - 2 = 0$

a = 5, b = 6, c = –2

$$x = \frac{-6 \pm \sqrt{6^2 - 4 \times 5 \times -2}}{2 \times 5}$$

$$x = \frac{-6 \pm \sqrt{76}}{10}$$

$x = 0.27$ or $x = -1.47$ (2 d.p.)

c) $(2x + 3)^2 = 15$

$4x^2 + 12x + 9 = 15$

$4x^2 + 12x - 6 = 0$

$2x^2 + 6x - 3 = 0$

a = 2, b = 6, c = –3

$$x = \frac{-6 \pm \sqrt{6^2 - 4 \times 2 \times (-3)}}{2 \times 2}$$

$$x = \frac{-6 \pm \sqrt{60}}{4}$$

$x = 0.44$ or $x = -3.44$ (2 d.p.)

Q12 a) $x^2 + 12x + 15 = 0$

$(x + 6)^2 - 36 + 15 = 0$

$(x + 6)^2 - 21 = 0$

$x = -6 \pm \sqrt{21}$

b) $x^2 - 6x - 2 = 0$

$(x - 3)^2 - 9 - 2 = 0$

$(x - 3)^2 - 11 = 0$

$x = 3 \pm \sqrt{11}$

Q13 If the graph has a turning point at (2, 5), then the completed square form of the equation is $y = (x - 2)^2 + 5 = x^2 - 4x + 4 + 5 = x^2 - 4x + 9$.
So $p = -4$ and $q = 9$.

Q14
$$\frac{2}{x+3} + \frac{1}{x-1} = \frac{2(x-1) + (x+3)}{(x+3)(x-1)}$$
$$= \frac{2x - 2 + x + 3}{(x+3)(x-1)}$$
$$= \frac{3x+1}{(x+3)(x-1)}$$

Q15 a)

n:	1	2	3	4
term:	7	9	11	13
Difference:	2	2	2	
2n:	2,	4,	6,	8

To get from 2n to the term, you have to add 5. So nth term = 2n + 5

b)

n:	1	2	3	4
term:	11	8	5	2
Difference:	–3	–3	–3	
–3n:	–3,	–6,	–9,	–12

To get from –3n to the term, you have to add 14. So nth term = –3n + 14

c) Sequence: 5 9 15 23

First difference: 4 6 8

Second difference: 2 2

Coefficient of $n^2 = 2 \div 2 = 1$.

Sequence given by n^2: 1 4 9 16

Actual sequence – n^2 sequence:

4 5 6 7

Difference: 1 1 1

So this is a linear sequence with nth term n + 3.

nth term: $n^2 + n + 3$.

Q16 $32 = n^2 + 7$

$n^2 = 25$

$n = 5$

So 32 is the 5th term in the sequence.

Q17 a) $4x + 3 \le 6x + 7$

$-2x \le 4$

$x \ge -2$

Remember to flip the inequality sign when you divide by a negative number.

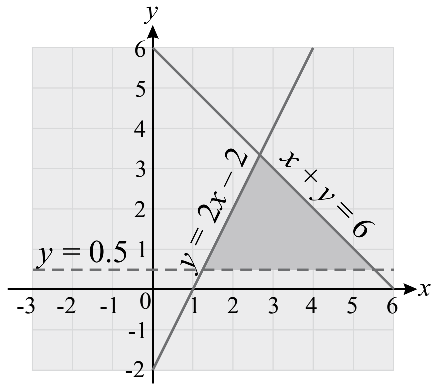
b) $5x^2 > 180$

$x^2 > 36$

If $x^2 = 36$, $x = 6$ or $x = -6$.

As $x^2 > 36$, $x < -6$ or $x > 6$.

Q18



Worked Answers

Q19 Put $x = 3$ and $x = 4$ into the equation:
$(3)^3 - 4(3)^2 + 2(3) - 3 = -6$
$(4)^3 - 4(4)^2 + 2(4) - 3 = 5$
There is a sign change, so there is a solution between $x = 3$ and $x = 4$.

Q20 $4x + 5y = 23$ (1)
$3y - x = 7$ (2)

(2) × 4: $-4x + 12y = 28$ (3)

(1) + (3): $17y = 51$
$y = 3$

Sub into (1): $4x + 5 \times 3 = 23$
$4x = 8$, so $x = 2$

Solution: $x = 2$, $y = 3$

Q21 $y = 3x + 4$ (1)
$x^2 + 2y = 0$ (2)
Substitute (1) into (2): $x^2 + 2(3x + 4) = 0$

$x^2 + 6x + 8 = 0$

$(x + 2)(x + 4) = 0$

$x = -2$ or $x = -4$

Substitute $x = -2$ into (1): $y = 3 \times (-2) + 4 = -2$
Substitute $x = -4$ into (1): $y = 3 \times (-4) + 4 = -8$
Solution: $x = -2$, $y = -2$ and $x = -4$, $y = -8$
You'd usually substitute the quadratic equation into the linear one, but it makes sense to do it the other way round here (but either method is fine here).

Q22 Take an even number, $2p$, and an odd number, $2q + 1$. Their product is $2p \times (2q + 1)$ $= 4pq + 2p = 2(2pq + p)$, which is even as $(2pq + p)$ is an integer.

Q23 **a)** $f(3) = (3)^2 - 3 = 9 - 3 = 6$
b) $g(4.5) = 4(4.5) = 18$
c) $fg(x) = f(4x) = (4x)^2 - 3 = 16x^2 - 3$
d) $x = f(y)$:
$x = y^2 - 3$
$x + 3 = y^2$
$y = \sqrt{x + 3}$
Then replace y with $f^{-1}(x)$:
$f^{-1}(x) = \sqrt{x + 3}$

Section Three — Graphs

Page 43 — Straight Lines and Gradients

Q1 E.g. line goes through points (–3, 20) and (1, 0)

$$\text{Gradient} = \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{0 - 20}{1 - (-3)} \textit{ [1 mark]} = \frac{-20}{4} = -5 \textit{ [1 mark]}$$

Page 44 — y = mx + c

Q1 y-intercept 'c' = 2
Find the gradient:

$$\text{Gradient} = \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{6 - 2}{6 - 0} = \frac{4}{6} = \frac{2}{3}$$

So $y = \frac{2}{3}x + 2$

[2 marks available — 1 mark for correct c, 1 mark for correct m]

Q2 Find the gradient:

$$\text{Gradient} = \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{7 - 5}{4 - 0} = \frac{2}{4} = \frac{1}{2} \textit{ [1 mark]}$$

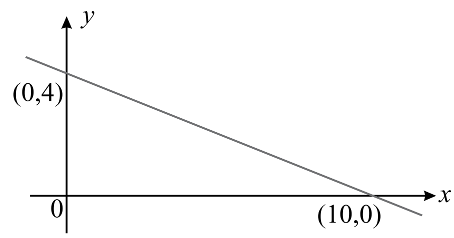
so $m = \frac{1}{2}$
Line passes through (0, 5) so y-intercept (c) is 5
[1 mark]

Therefore, equation is $y = \frac{1}{2}x + 5$ ***[1 mark]***

Page 45 — Drawing Straight-Line Graphs

Q1 When $x = 0$, $5y = 20$
$y = 4$, so line crosses at (0, 4)
[1 mark]

When $y = 0$, $2x = 20$
$x = 10$, so line crosses at (10, 0)
[1 mark]



[1 mark]

Page 46 — Coordinates and Ratio

Q1 Centre of circle = midpoint of AB:

$$\left(\frac{-4 + 8}{2}, \frac{-1 + -3}{2}\right) \textit{ [1 mark]}$$

$$= \left(\frac{4}{2}, \frac{-4}{2}\right) = (2, -2) \textit{ [1 mark]}$$

Worked Answers

Q2 Find the coordinates of point P:
At point P $y = 0$, so $3x + 6 = 0$
$3x = -6$
$x = -2$
So point P is at $(-2, 0)$. ***[1 mark]***
Find the coordinates of point Q:
At point Q $x = 0$, so $y = 6$
So point Q is at $(0, 6)$. ***[1 mark]***
Find coordinates of point R.
Difference in x between P and Q: $0 - -2 = 2$
Difference in y between P and Q: $6 - 0 = 6$
The ratio tells you QR is twice as big as PQ, so double the distances in x and y between P and Q
Difference in x between Q and R: $2 \times 2 = 4$
Difference in y between Q and R: $6 \times 2 = 12$
[1 mark]
x coordinate of R: $0 + 4 = 4$
y coordinate of R: $6 + 12 = 18$
So point R is at $(4, 18)$. ***[1 mark]***

Page 47 — Parallel and Perpendicular Lines

Q1 Rearrange given equation into $y = \mathrm{m}x + \mathrm{c}$ form to find m (gradient):
$2x + 2y = 3 \Rightarrow y = -x + 1.5$,
so $\mathrm{m} = -1$ ***[1 mark]***
Equation of parallel line is $y = -x + \mathrm{c}$.
The line passes through (1, 4), so substitute
$x = 1$ and $y = 4$: $4 = -1 + \mathrm{c} \Rightarrow \mathrm{c} = 5$ ***[1 mark]***
Equation: $y = -x + 5$ ***[1 mark]***

Q2 Rearrange given equations into $y = \mathrm{m}x + \mathrm{c}$ form to find gradients.
Line 1: $y + 5x = 2$
$y = -5x + 2$
Gradient of line 1 $= -5$ ***[1 mark]***
Line 2: $5y = x + 3$
$y = \frac{1}{5}x + \frac{3}{5}$
Gradient of line 2 $= \frac{1}{5}$ ***[1 mark]***
$-5 \times \frac{1}{5} = -1$
So $y + 5x = 2$ and $5y = x + 3$ are perpendicular as their gradients multiply together to give -1.
[1 mark]

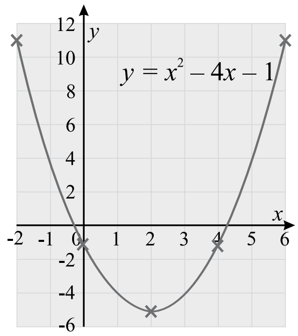
Page 48 — Quadratic Graphs

Q1 Make a table of values, e.g.

x	−2	0	2	4	6
y	11	−1	−5	−1	11

[1 mark for a table containing two or three correct points. 2 marks for a table containing at least four correct points.]

Plot the points from the table and draw a smooth curve through them.



[1 mark for at least four points correctly plotted and 1 mark for a smooth curve through the points.]

Quadratic graphs are known as 'bucket-shaped'. But it'd make a rubbish bucket as it'd topple over.

Page 49 — Harder Graphs 1

Q1 Find change in y and change in x:
change in $y = 12 - 0 = 12$
change in $x = 5 - 0 = 5$
Use Pythagoras' theorem to find the radius:
$a^2 + b^2 = c^2$
$5^2 + 12^2 = c^2$ ***[1 mark]***
$c^2 = 169$
$c = 13$
So the radius is 13 units. ***[1 mark]***
Equation of the circle is $x^2 + y^2 = 169$ ***[1 mark]***

Page 50 — Harder Graphs 2

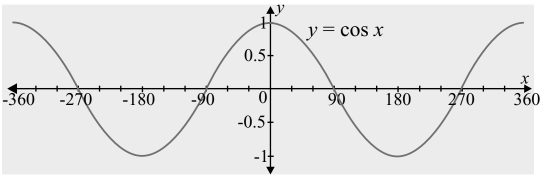
Q1 **a)** $P = ab^t$
When $t = 0$, $P = 16$. Sub into equation:
$16 = ab^0$ ***[1 mark]***
$16 = a \times 1$
$a = 16$ ***[1 mark]***
When $t = 4$, $P = 256$. Sub into equation:
$256 = 16 \times b^4$ ***[1 mark]***
$16 = b^4$
$b = 2$ ***[1 mark]***

b) $P = 16 \times 2^7$ ***[1 mark]***
$P = 2048$ ***[1 mark]***

Worked Answers

Page 51 — Harder Graphs 3

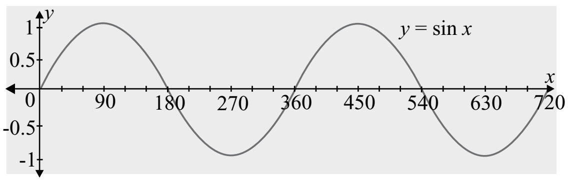
Q1 a)



[1 mark for peaks at y = 1 and troughs at y = –1. 1 mark for cos 'bucket' between x = 0° and x = 360º. 1 mark for 'bucket' repeated between x = –360° and x = 0°. 1 mark for correct intersections with x-axis.]

The key to drawing the cos graph is to draw the 'bucket' between 0° and 360°, then repeat it in the negative direction.

b)

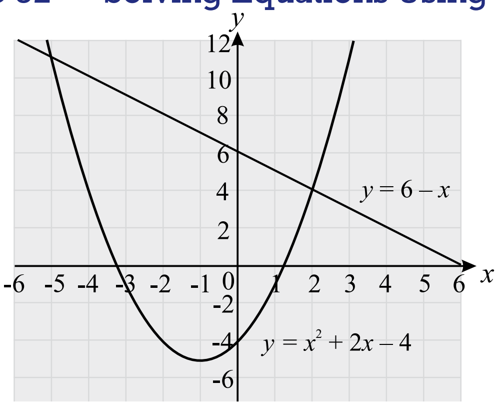


[1 mark for peaks at y = 1 and troughs at y = –1. 1 mark for sine 'wave' between x = 0° and x = 360°. 1 mark for 'wave' repeated between x = 360° and x = 720°. 1 mark for correct intersections with x-axis.]

The key to drawing the sin graph is to draw the 'wave' between 0° and 360°, then repeat it. Remember the peak is at 90° and trough at 270°.

Page 52 — Solving Equations Using Graphs

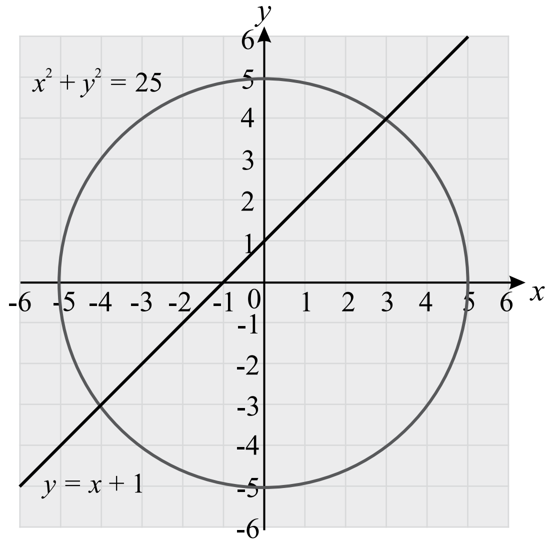
Q1



$x = 2$, $y = 4$ and $x = –5$, $y = 11$

[4 marks available — 1 mark for plotting each line correctly, 1 mark for each correct answer.]

Q2



$x = –4$, $y = –3$ and $x = 3$, $y = 4$

[4 marks available — 1 mark for plotting each graph correctly, 1 mark for each correct solution.]

Page 53 — Graph Transformations

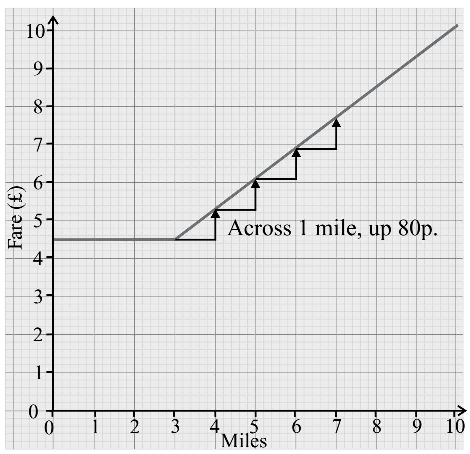
Q1 **a)** This is a reflection in the y-axis, so the x-coordinate is multiplied by –1: (4, 3) ⇒ (–4, 3) ***[1 mark]***

b) This is a y-shift 4 units downwards, so the y-coordinate of the maximum point decreases by 4: (4, 3) ⇒ (4, –1) ***[1 mark]***

c) This is an x-shift by 2 to the right and a y-shift upwards by 1, so increase x by 2 and increase y by 1: (4, 3) ⇒ (6, 4) ***[1 mark]***

Page 54 — Real-Life Graphs

Q1 The fare for any distance between 0 and 3 miles is £4.50, so the first part of the graph is horizontal, as shown below. Then the graph goes up 80p for each mile:



Worked Answers

[1 mark for suitable scale and axes labels, 1 mark for initial horizontal line, 1 mark for sloped line with correct gradient, 1 mark for continuing graph to 10 miles.]

Rather than adding steps of 80p, it's easier to work out the cost for 10 miles and join this point to the cost at 3 miles with a straight line.

Page 55 — Distance-Time Graphs

Q1 **a)** The horizontal section on the graph started at 8:45 and finished at 9:00. This is 15 minutes. ***[1 mark]***

b) Gradient of the graph from 9:00 to 9:30:
30 minutes = 0.5 hours ***[1 mark]***

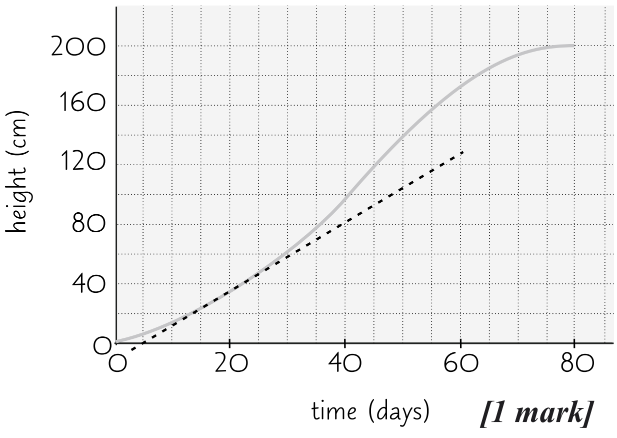
$\frac{18-12}{0.5} = 12$ km/h ***[1 mark]***

Page 56 — Velocity-Time Graphs

Q1 Find the total area under the graph:
Area 1 (triangle): $0.5 \times 7.5 \times 35 = 131.25$
Area 2 (rectangle): $7.5 \times 35 = 262.5$
Area 3 (trapezium): $0.5 \times (35 + 45) \times 12.5 = 500$
Area 4 (triangle): $0.5 \times 45 \times 5 = 112.5$
$131.25 + 262.5 + 500 + 112.5 = 1006.25$ m
[3 marks for correct answer, otherwise 2 marks for calculating all individual areas correctly, or 1 mark for calculating 2 areas correctly]

Page 57 — Gradients of Real-Life Graphs

Q1



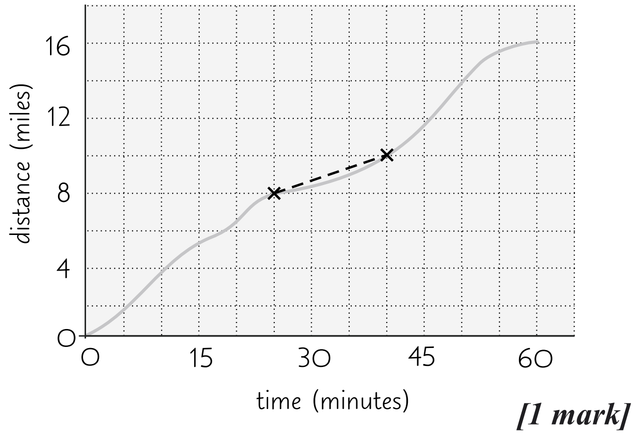
[1 mark]

E.g. Gradient $= \frac{\text{change in } y}{\text{change in } x} = \frac{80-0}{40-5} = \frac{80}{35}$

≈ 2.3 cm per day ***[1 mark]***

If you get an answer between 2.0 and 2.6 you'll still get the marks — your answer will depend on the line you've drawn and the points you've chosen.

Q2



[1 mark]

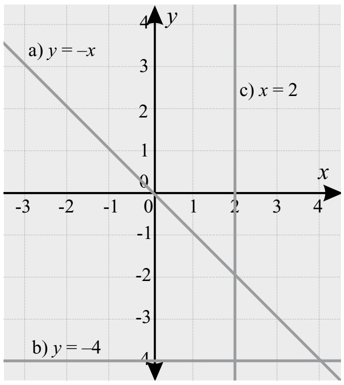
Gradient $= \frac{\text{change in } y}{\text{change in } x} = \frac{10-8}{40-25}$

$= \frac{2}{15}$ miles per minute ***[1 mark]***

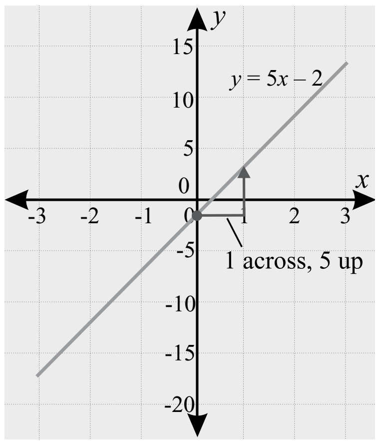
$= 8$ miles per hour

Page 58 — Revision Questions

Q1



Q2 Rearrange equation into '$y = mx + c$' form:
$5x = 2 + y \Rightarrow y = 5x - 2$
y-intercept (c) = –2, gradient (m) = 5



Q3 Line goes through points (0, 10) and (10, 30)

Gradient $= \frac{\text{change in } y}{\text{change in } x} = \frac{30-10}{10-0} = \frac{20}{10} = 2$

So m = 2. The y-intercept (c) = 10, so equation is $y = 2x + 10$.

Worked Answers

Q4 Gradient $= \dfrac{\text{change in } y}{\text{change in } x} = \dfrac{-3 - -6}{6 - 3} = \dfrac{3}{3} = 1$

Find equation of the line: $y = mx + c$

$y = x + c$

Substitute in $(3, -6)$ $\quad -6 = 3 + c$

$c = -9$

$y = x - 9$

Q5 Line is perpendicular to $y = 2x - 1$,

so gradient $= -1 \div 2 = -\dfrac{1}{2}$.

It passes through $(4, 2)$, so substitute $x = 4$ and $y = 2$ into $y = -\dfrac{1}{2}x + c$ to find c.

$2 = -\dfrac{1}{2}(4) + c \;\Rightarrow\; 2 = -2 + c \;\Rightarrow\; c = 4$

Equation of line is $y = -\dfrac{1}{2}x + 4$

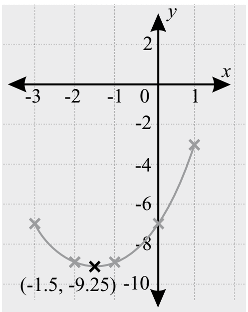
Q6 **a)**

x	–3	–2	–1	0	1
y	–7	–9	–9	–7	–3

b) Use the table of values and symmetry to find the minimum point:
x-coordinate lies in centre of –2 and –1, so $x = -1.5$. To find the y-coordinate put $x = -1.5$ back into $y = x^2 + 3x - 7$

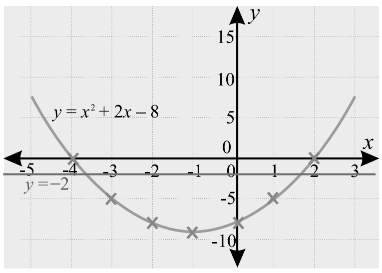
$= (-1.5)^2 + 3(-1.5) - 7$

$= -9.25$



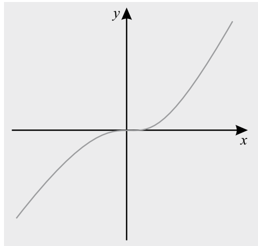
Q7

x	–4	–3	–2	–1	0	1	2
y	0	–5	–8	–9	–8	–5	0

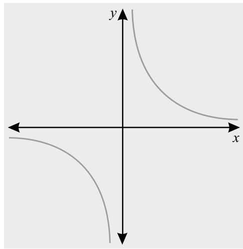


The equation $-2 = x^2 + 2x - 8$ is what you get when you put $y = -2$ into the equation of the graph above. Draw the line $y = -2$, then estimate the x-values where the curve crosses the line: $x = -3.6$ (allow between –3.8 and –3.4) or 1.6 (allow between 1.4 and 1.8).

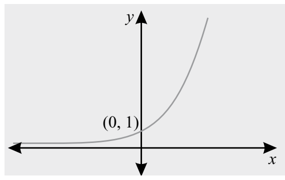
Q8 **a)** A graph with a "wiggle" in the middle. E.g.



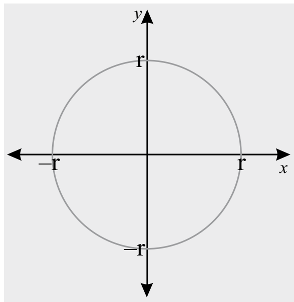
b) A graph made up of two curves in diagonally opposite quadrants. The curves are symmetrical about the lines $y = x$ and $y = -x$.
E.g.



c) A graph which curves rapidly upwards. E.g.



d) A circle with radius r, centre (0, 0). E.g.



Q9 Substitute the pairs of x and y values into the equation $y = bc^x$ then solve for b and c:

$16 = bc^2$ – Eq 1

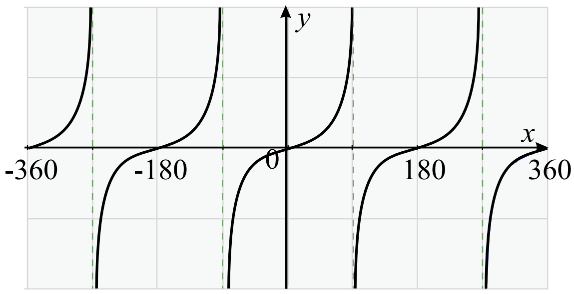
$128 = bc^3$ – Eq 2

Eq 2 ÷ Eq 1: $128 \div 16 = bc^3 \div bc^2 \;\Rightarrow\; c = 8$

Substitute c = 8 into Eq 1:

$16 = b \times 8^2 \;\Rightarrow\; b = 0.25$

Q10



Worked Answers

Q11 Write both equations in the form $y = mx + c$:

Line 1: $4y - 2x = 32$

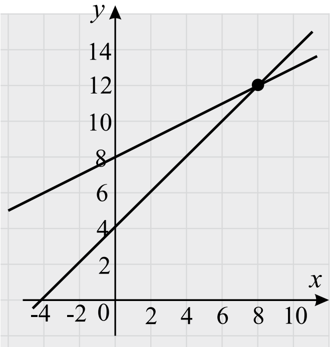
$4y = 2x + 32$

$y = \frac{1}{2}x + 8$

Line 2: $3y - 12 = 3x$

$3y = 3x + 12$

$y = x + 4$



$x = 8$, $y = 12$

Q12 $x^2 + 4x = 0$

$x^2 + 4x + 6 = 6$

$x^2 - x + 6 = -5x + 6$

$-x + 6 = -x^2 - 5x + 6$

The equation of the line you would need to draw is $y = -x + 6$.

Q13 Translation on y-axis: $y = f(x) + a$

Translation on x-axis: $y = f(x - a)$

Reflection: $y = -f(x)$ or $y = f(-x)$, where $y = -f(x)$ is reflected in the x-axis and $y = f(-x)$ is reflected in the y-axis.

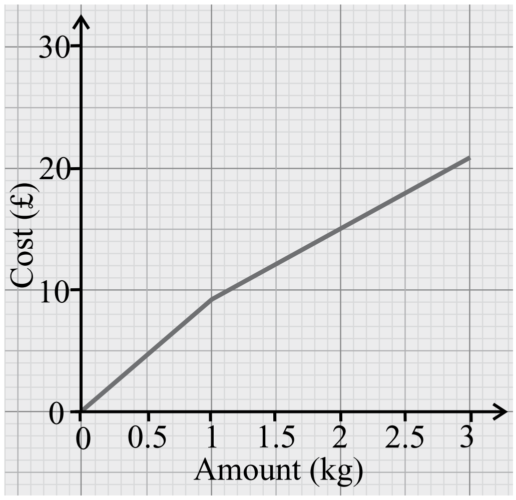
Q14 **a)** $y = (-x)^3 + 1$ is the original graph reflected in y-axis.

b) $y = (x + 2)^3 + 1$ is the original graph translated by 2 units in the negative x-direction.

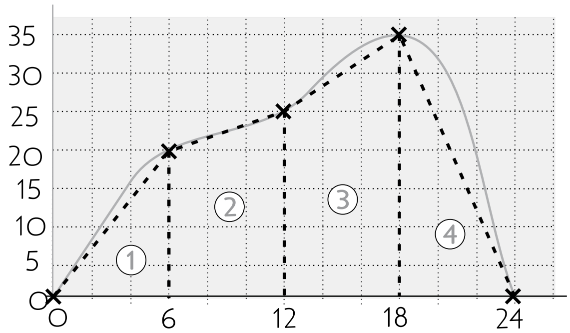
c) $y = x^3 + 4$ is the original graph translated upwards by 3 units.

d) $y = x^3 - 1$ is the original graph translated downwards by 2 units.

Q15 The first kg cost 90p per 100 g, so 1 kg costs £9, so the graph will go through (1, 9). Each extra kg costs 60p per 100 g, which is £6 per kg. So 3 kg costs £9 + £6 + £6 = £21, so the graph finishes at (3, 21).



Q16 **a)**



Find the area of each trapezium/triangle:

(1): $\frac{1}{2} \times 20 \times 6 = 60$

(2): $\frac{1}{2} \times (20 + 25) \times 6 = 135$

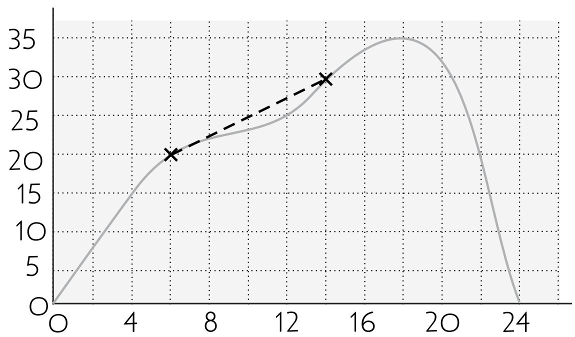
(3): $\frac{1}{2} \times (25 + 35) \times 6 = 180$

(4): $\frac{1}{2} \times 35 \times 6 = 105$

Total area = 60 + 135 + 180 + 105 = 480

So the sledge travelled around 480 metres.

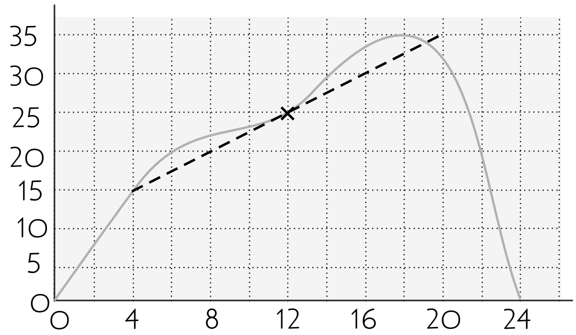
b) Draw a straight line connecting the points.



Gradient $= \frac{30 - 20}{14 - 6} = \frac{10}{8} = 1.25\ \text{m/s}^2$

Worked Answers

c) Draw a tangent to the curve at $x = 12$



Gradient $= \frac{30-20}{16-8} = \frac{10}{8} = 1.25\ \text{m/s}^2$

Section Four — Ratio, Proportion and Rates of Change

Page 61 — Ratios

Q1 **a)** $25:35$
$= (25 \div 5):(35 \div 5)$
$= 5:7$ ***[1 mark]***

b) $3.4:5.1$
$= 34:51$
$= (34 \div 17):(51 \div 17)$
$= 2:3$ ***[1 mark]***

c) $\frac{9}{4} : \frac{15}{2}$
$= (\frac{9}{4} \times 4):(\frac{15}{2} \times 4)$ ***[1 mark]***
$= 9:30$
$= (9 \div 3):(30 \div 3)$
$= 3:10$ ***[1 mark]***

Q2 **a)** Total parts $= 11 + 2 = 13$
The fraction of the squash made up from concentrate is $\frac{2}{13}$ ***[1 mark]***

b) 13 parts = 1.95 litres
1 part = 1.95 ÷ 13 = 0.15 litres ***[1 mark]***
11 parts = 0.15 × 11 = 1.65 litres ***[1 mark]***

Q3 8 parts – 3 parts = 5 parts
5 parts = 25 years
1 part = 25 ÷ 5 = 5 years ***[1 mark]***
7 parts = 5 × 7 = 35 years
So Graham is 35 years old ***[1 mark]***

Q4 Call the original number of red balls r and the original number of blue balls b.
Then:
$r - 2 : b - 2 = 5:7$ and $r + 7 : b + 7 = 4:5$
[1 mark]

$\frac{r-2}{b-2} = \frac{5}{7}$ and $\frac{r+7}{b+7} = \frac{4}{5}$ ***[1 mark]***

So $7(r - 2) = 5(b - 2)$ and $5(r + 7) = 4(b + 7)$
Solve the equations simultaneously:
$7r - 5b = 4$ [1] and $5r - 4b = -7$ [2] ***[1 mark]***
[1] × 4: $28r - 20b = 16$ [3]
[2] × 5: $25r - 20b = -35$ [4]
[3] – [4]: $3r = 51$
$r = 17$ ***[1 mark]***
Sub into [1]: $(7 \times 17) - 5b = 4$ ***[1 mark]***
$5b = 115$, so $b = 23$ ***[1 mark]***
Solution: $r = 17$, $b = 23$

An equally correct method would be to first eliminate r in order to find b and then to substitute the value of b back into one of the equations to find r.

Page 62 — Direct and Inverse Proportion

Q1 1 carpenter would take 4 × 2 = 8 hours to make 3 bookcases ***[1 mark]***
and 8 ÷ 3 = 2.666... hours to make 1 bookcase.
[1 mark]
5 carpenters would take
2.666... ÷ 5 = 0.533... hours to make 1 bookcase
[1 mark]
so they'd take 0.533... × 10 = 5.33... hours = 5 hours 20 mins to make 10 bookcases.
[1 mark]

Q2 For 8 people to go on the rollercoaster 1 time:
£43.20 ÷ 6 = £7.20 ***[1 mark]***
For 8 people to go on the rollercoaster 5 times:
£7.20 × 5 = £36 ***[1 mark]***
For 1 person to go on the rollercoaster 5 times:
£36 ÷ 8 = £4.50 ***[1 mark]***
For 15 people to go on the rollercoaster 5 times:
£4.50 × 15 = £67.50 ***[1 mark]***

You could also find the cost for 1 person to go on the rollercoaster 1 time = £43.20 ÷ 8 ÷ 6 = £0.90.
Then multiply: £0.90 × 15 × 5 = £67.50.

Worked Answers

Page 63 — Direct and Inverse Proportion

Q1 $V \propto t$ so $V = kt$ ***[1 mark]***
If $V = 105$ m/s when $t = 5$ seconds
$105 = 5k$
$k = 21$ ***[1 mark]***
$V = 21t$ so when $t = 13$ seconds,
$V = 21 \times 13 = 273$ m/s ***[1 mark]***

Q2 $P \propto \frac{1}{Q^2}$ so $P = \frac{k}{Q^2}$ ***[1 mark]***
If $P = 3$ when $Q = 4$
$3 = \frac{k}{4^2}$ ***[1 mark]***
$k = 3 \times 4^2 = 3 \times 16 = 48$ ***[1 mark]***
so $P = \frac{48}{Q^2}$
When $P = 8$:
$8 = \frac{48}{Q^2}$
$Q^2 = 48 \div 8 = 6$
$Q = \sqrt{6}$ ***[1 mark]***

Page 64 — Percentages

Q1 30% of 250 = 0.3 × 250 = 75 runs ***[1 mark]***
$\frac{105}{75} \times 100$ ***[1 mark]*** = 140% ***[1 mark]***

Page 65 — Percentages

Q1 1.2 kg = 1.2 × 1000 = 1200 g ***[1 mark]***
1200 g – 900 g = 300 g less ***[1 mark]***
$\frac{300}{1200} \times 100 = 25\%$ less ***[1 mark]***

Q2 Selling at a 20% loss means that the selling price is 80% of the cost to make a kebab
£4.88 = 80% of cost price
1% of cost price = 4.88 ÷ 80 = 0.061 ***[1 mark]***
100% = 0.061 × 100 = £6.10 ***[1 mark]***
For a 10% loss, selling price is 90% of £6.10
= £6.10 × 0.9 = £5.49 ***[1 mark]***

Page 66 — Percentages

Q1 Each year the percentage increase in the value of the investment will be 4% of the original investment. Because the interest is simple, the same amount is added each time so the total percentage increase is just 5 lots of 4%.
So, in 5 years the percentage increase will be 4% × 5 ***[1 mark]*** = 20% ***[1 mark]***

Q2 Percentage of editors who are female and have a maths degree:
30% of 60% = 0.3 × 60% = 18% ***[1 mark]***
Percentage of editors who are male and have a maths degree:
100% – 60% = 40% male editors
20% of 40% = 0.2 × 40% = 8% ***[1 mark]***
Percentage of editors with a maths degree:
18% + 8% = 26% ***[1 mark]***

Page 67 — Compound Growth and Decay

Q1 Pippa will have $N = N_0(\text{multiplier})^n$
where N_0 = initial amount = £3200
multiplier = 1 + (2.5 ÷ 100) = 1.025
n = number of years = 3
So $N = 3200(1.025)^3$ ***[1 mark]***
= £3446.05 ***[1 mark]***
Kyle will have:
3% of £3200 = £3200 × 0.03
= £96 extra each year ***[1 mark]***
£96 × 3 = £288 so he has
£3200 + £288 = £3488
£3488 – £3446.05 = £41.95 so Kyle will have £41.95 more after 3 years. ***[1 mark]***

Page 68 — Unit Conversions

Q1 She drives 18 km × 2 = 36 km each day,
so 36 km × 5 = 180 km each week. ***[1 mark]***
180 km ≈ 180 ÷ 1.6 = 112.5 miles ***[1 mark]***
At 28 mpg, she will use
112.5 ÷ 28 = 4.0178... gallons of fuel ***[1 mark]***
4.0178... gallons ≈ 4.0178... × 4.5
= 18.0803... litres ***[1 mark]***
18.0803... litres cost:
18.0803... × 1.39 = 25.1316...
= £25 (to 2 s.f.) ***[1 mark]***

Page 69 — Speed, Density and Pressure

Q1 Volume of cone $= \frac{1}{3} \times \pi r^2 \times h_v$
$= \frac{1}{3} \times \pi \times 20^2 \times 60$
$= 25132.74...$ cm^3 ***[1 mark]***
Mass = density × volume
= 11.34 × 25132.74... ***[1 mark]***
= 285 005.2... g
= (285 005.2... ÷ 1000) kg
= 285.005... kg
= 285 kg (3 s.f.) ***[1 mark]***

Worked Answers

Page 70 — Revision Questions

Q1 There are $\frac{13}{8}$ or 1.625 times more pencils than rubbers.

Q2 **a)** $1.2:1.6$
$= (1.2 \times 10):(1.6 \times 10)$
$= 12:16$
$= 3:4$

b) $49\text{ g}:14\text{ g}$
$= (49\text{ g} \div 14):(14\text{ g} \div 14)$
$= 3.5\text{ g}:1\text{ g}$
$= 3.5:1$

Q3 Multiply by 30 to get 150 on the left-hand side:
$5:8 = (5 \times 30):(8 \times 30)$
$= 150:240$
So Sarah should order 240 blue scarves.

Q4 **a)** $5 + 8 + 12 = 25$ parts in total
Ryan delivers 5 parts $= \frac{5}{25} = \frac{1}{5}$

b) 25 parts = 800
1 part = $800 \div 25 = 32$
Krupa delivers 12 parts – 8 parts = 4 parts more than Joel.
4 parts = $32 \times 4 = 128$

Q5 44 oak trees and $(44 \div 2) \times 5 = 110$ pine trees
Let x be the number they planted:
$44 + x : 110 + x = 9:20$

$$\frac{44 + x}{110 + x} = \frac{9}{20}$$

$20(44 + x) = 9(110 + x)$
$880 + 20x = 990 + 9x$
$11x = 110$
$x = 10$
So 10 of each tree were planted.

Q6 $x:y = 4:1$ and $x - 6 : y - 6 = 10:1$
$\frac{x}{y} = \frac{4}{1}$ and $\frac{x-6}{y-6} = \frac{10}{1}$
$x = 4y$ and $x - 6 = 10(y - 6)$
$x = 4y$ [1] and $x - 10y = -54$ [2]
Substitute [1] into [2]:
$4y - 10y = -54$
$-6y = -54$
$y = 9$
$x = 4 \times 9 = 36$

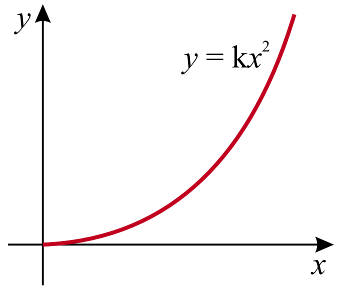
Q7 **a)** In 1 hour, 6 gardeners could plant:
$360 \div 3 = 120$ flowers
In 6 hours, 6 gardeners could plant:
$120 \times 6 = 720$ flowers
In 6 hours, 1 gardener could plant:
$720 \div 6 = 120$ flowers
In 6 hours, 8 gardeners could plant:
$120 \times 8 = 960$ flowers

You could also find that 1 gardener could plant $360 \div 3 \div 6 = 20$ flowers in an hour, and then multiply by 6 and 8 to find the final answer.

b) 6 gardeners to plant 1 flower takes:
$3 \div 360 = 0.008333...$ hours
6 gardeners to plant 1170 flowers takes:
$0.008333... \times 1170 = 9.75$ hours
1 gardener to plant 1170 flowers takes:
$9.75 \times 6 = 58.5$ hours
15 gardeners to plant 1170 flowers takes:
$58.5 \div 15 = 3.9$ hours

Alternatively, if you'd found the number of flowers one gardener could plant in an hour (20) in part a), you could do $1170 \div 15 \div 20 = 3.9$.

Q8 **a)** $y \propto x^2 \Rightarrow y = kx^2$

b)


Q9 If the pressure is p and the side length s, then:
$p \propto \frac{1}{s^2} \Rightarrow p = \frac{k}{s^2}$
When $s = 3$ cm, $p = 17$ Pa, so:
$17 = \frac{k}{3^2}$
$k = 17 \times 3^2 = 153$
$p = \frac{153}{s^2}$ so when s = 13,
$p = \frac{153}{13^2} = 0.91$ Pa (2 d.p.)

Q10 **a)** 20% of 95 = $0.2 \times 95 = 19$

b) Increase 20 by 95%:
$20 \times 1.95 = 39$

c) 20 as a percentage of 95:
$(20 \div 95) \times 100 = 21.05\%$ (2 d.p.)

d) 95 as a percentage of 20:
$(95 \div 20) \times 100 = 475\%$

Q11 percentage change = (change ÷ original) × 100

Q12 £800 – £520 = £280 decrease
$(280 \div 800) \times 100 = 35\%$ decrease

Q13 20.24 m = 115%
0.176 m = 1%
17.6 m = 100%
So the original height was 17.6 metres.

Q14 Find 8% of 25%:
$0.08 \times 25\% = 2\%$

Q15 $N = N_0(\text{multiplier})^n$

Worked Answers

Q16 a) N = $80(1.07)^{10}$
= £157.37 (to the nearest penny)

b) Use trial and error.
If n = 13, N = $80(1.07)^{13}$ = 192.7876
If n = 14, N = $80(1.07)^{14}$ = 206.282732
So it will be 14 years before the card is worth over £200.

Q17 a) 5.6 litres = (5.6 × 1000) cm^3 = 5600 cm^3

b) 1 foot ≈ 30 cm
8 feet ≈ (8 × 30) cm = 240 cm

c) 3 m/s = (3 ÷ 1000) km/s = 0.003 km/s
0.003 km/s = (0.003 × 3600) km/h
= 10.8 km/h

d) 12 m^3 = (12 × 100 × 100 × 100) cm^3
= 12 000 000 cm^3

e) 1280 mm^2 = (1280 ÷ 10 ÷ 10) cm^2
= 12.8 cm^2

f) 2.75 cm^3 = (2.75 × 10 × 10 × 10) mm^3
= 2750 mm^3

Q18 Speed = D ÷ T = 63 ÷ 1.5 = 42 mph

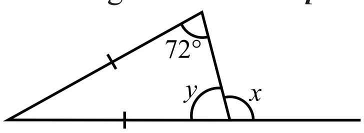
Q19 5 kg = 5 × 1000 g = 5000 g
V = M ÷ D = 5000 ÷ 0.4 = 12 500 cm^3

Q20 A = F ÷ P = 1320 ÷ 120 = 11 m^2

Section Five — Geometry and Measures

Page 71 — Geometry

Q1 Angle y on the diagram below is 72°, because the triangle is isosceles ***[1 mark]***.



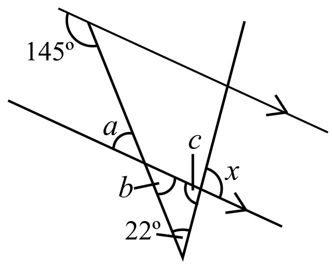
Angles on a straight line add up to 180°, so x = 180° – 72° = 108° ***[1 mark]***.

Page 72 — Parallel Lines

Q1 Allied angles add up to 180°, so
x – 15° + 3x + 75° = 180° ***[1 mark]***
4x + 60° = 180°
4x = 120° ***[1 mark]***
x = 30° ***[1 mark]***

Page 73 — Geometry Problems

Q1 One method is:
Find angle a (see diagram below) using allied angles:
a = 180° – 145° = 35°
Using vertically opposite angles:
b = a = 35°
Using sum of angles in a triangle:
c = 180° – (22° + 35°) = 123°
And finally, using vertically opposite angles:
x = c = 123°



[3 marks available. Full marks for correct answer, otherwise 1 mark for each other angle determined, up to a maximum of 2 marks.]

Page 74 — Polygons

Q1 Exterior angle of regular decagon
= 360° ÷ 10 = 36° ***[1 mark]***
Interior angle = 180° – 36° = 144° ***[1 mark]***
An alternative method is to find the sum of the interior angles of a decagon using the formula (n – 2) × 180° (1440°), and divide this by 10 to find the size of each interior angle.

Page 77 — Circle Geometry

Q1 Using the alternate segment theorem gives you both of the required angles. Alternatively, you can find one of them using the alternate segment theorem and then the other using the 'angles in the same segment' theorem.
angle ABD = 63° ***[1 mark]***
angle ACD = 63° ***[1 mark]***

Page 78 — Congruent Shapes

Q1 E.g. Angles ABD and BDC are the same (alternate angles) ***[1 mark]***.
Angles ADB and DBC are the same (alternate angles) ***[1 mark]***.
Side BD (the side opposite the obtuse angle in each triangle) is the same length in each shape ***[1 mark]***.
So triangles ABD and BCD are congruent as the condition AAS holds.

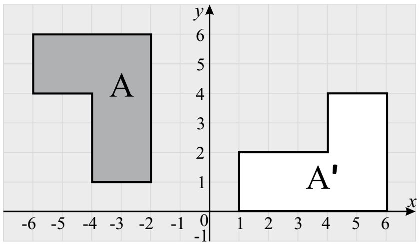
Worked Answers

Page 79 — Similar Shapes

Q1 Triangles ABC and ADE are similar.
Scale factor = 18 ÷ 12 = 1.5 ***[1 mark]***
AD = 20 × 1.5 = 30 cm ***[1 mark]***
DB = AD – AB = 30 – 20 = 10 cm ***[1 mark]***
You have to pick out the corresponding sides first.
Do this by matching the angles up.

Page 80 — The Four Transformations

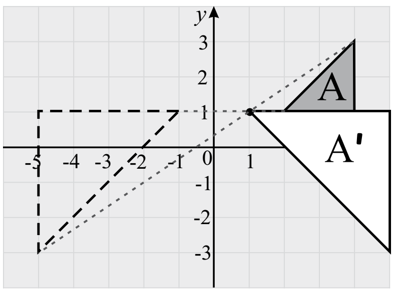
Q1



[2 marks for a shape with vertices at (1, 0), (6, 0), (6, 4), (4, 4), (4, 2) and (1, 2). 1 mark for a rotation 90° clockwise about the wrong centre of rotation.]

Page 81 — The Four Transformations

Q1 The scale factor is –2, so multiply the distance of each vertex from the centre of enlargement by 2, and measure this distance coming out the other side of the centre of enlargement.
Then reflect the enlarged shape in the y-axis (the line $x = 0$).



[3 marks for a shape with vertices at (1, 1), (5, 1) and (5, –3), otherwise 1 mark for an enlargement by an incorrect negative scale factor or by a positive scale factor of 2, 1 mark for a reflection in the y-axis.]

Page 82 — Area — Triangles and Quadrilaterals

Q1 Area of triangle = ½ (5 × 16) = 40 cm²
Area of rectangle = $4x$ cm² ***[1 mark for both]***
$40 = 4x$
$x = 10$ ***[1 mark]***

Page 83 — Area — Circles

Q1 a) Area of sector = $\frac{x}{360} \times \pi r^2$
$= \frac{150}{360} \times \pi \times 8^2$ ***[1 mark]***
= 83.78 cm² (2 d.p.) ***[1 mark]***

b) Arc length = $\frac{x}{360} \times 2\pi r$
$= \frac{150}{360} \times 2\pi \times 8$ ***[1 mark]***
= 20.94 cm (2 d.p.) ***[1 mark]***

c) Area of triangle = $\frac{1}{2}ab\sin x$
$= \frac{1}{2} \times 8 \times 8 \times \sin 150° = 16$ cm² ***[1 mark]***
Area of segment
= area of sector – area of triangle
= 83.775... – 16 = 67.78 cm² (2 d.p.)
[1 mark]

Page 84 — 3D Shapes — Surface Area

Q1 Surface area of a cone = $\pi rl + \pi r^2$
$125\pi = 5\pi l$ ***[1 mark]*** $+ 5^2\pi$ ***[1 mark]***
$5l = 100$
$l = 20$ cm ***[1 mark]***

Page 86 — 3D Shapes — Volume

Q1 Volume of sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 9^3$
$= 972\pi$ ***[1 mark]***
Volume of cone = $\frac{1}{3} \times \pi r^2 \times h_v = \frac{1}{3} \times \pi \times 9^2 \times h$
$= 27\pi h$ ***[1 mark]***
$972\pi = 27\pi h$ ***[1 mark]***
$h = 36$ cm ***[1 mark]***

Q2 Volume of pyramid = $\frac{1}{3}$ × base area × height
$= \frac{1}{3} \times 60^2 \times 110 = 132\ 000$ cm³
0.1 litres per second = 100 cm³ per second, so it will take 132 000 ÷ 100 = 1320 seconds to fill.
1320 seconds = 22 minutes, so it will take longer than 20 minutes to fill the pyramid.
[4 marks available — 1 mark for finding the volume of the pyramid, 1 mark for converting the rate of flow into cm³ (or converting the volume into litres), 1 mark for finding the time taken to fill the pyramid, 1 mark for conclusion.]
You could have done this one by working out how much water will go into the pyramid in 20 minutes and compared this to the volume of the pyramid to see if it was full.

Worked Answers

Page 87 — More Enlargements and Projections

Q1 You enlarge the middle doll's height by a scale factor of $\frac{1}{2}$ to get the smallest doll's height. So the smallest doll's surface area is the middle doll's surface area multiplied by $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$ ***[1 mark]***.

Smallest doll's surface area = $\frac{1}{4} \times 80 = 20$ cm^2 ***[1 mark]***

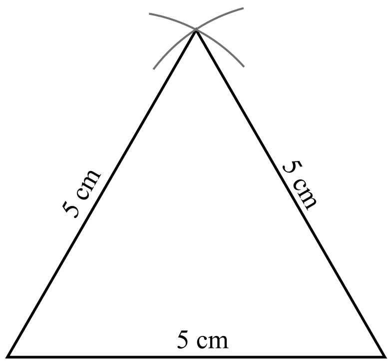
You enlarge the largest doll's height by a scale factor of $\frac{1}{3}$ to get the smallest doll's height. So the smallest doll's volume is the largest doll's volume multiplied by $\left(\frac{1}{3}\right)^3 = \frac{1}{27}$ ***[1 mark]***.

Smallest doll's volume = $\frac{1}{27} \times 216 = 8$ cm^3 ***[1 mark]***

Interesting fact: the world-record breaking set of Russian dolls contains 51 dolls. The smallest doll is just over 3 mm tall.

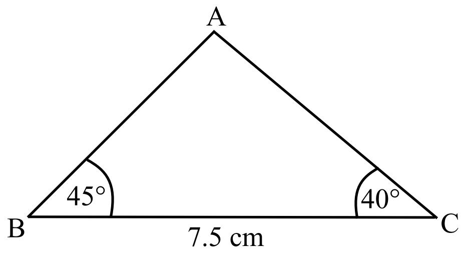
Page 88 — Triangle Construction

Q1 First draw a base line of 5 cm, then use compasses set to 5 cm to draw an arc from each end of the base line. Where the arcs cross is the tip of the triangle.



[1 mark for equilateral triangle with sides within 1 mm of 5 cm. 1 mark for visible construction marks.]

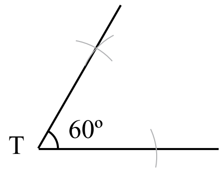
Q2 First draw a base line of 7.5 cm and label it BC. Next use a protractor to measure a 45° angle at B and a 40° angle at C. The point where the lines drawn at these angles meet is the tip of the triangle.



[1 mark for triangle with its longest side within 1 mm of 7.5 cm and angles within 1° of correct size. 1 mark for correct labels.]

Page 90 — Loci and Construction

Q1 Follow the method at the top of page 90.

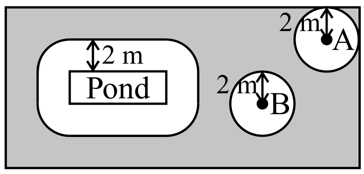


[1 mark for 60° angle (within 1°). 1 mark for correct construction lines visible.]

Don't be too dainty with your compass arcs — the examiners need to see them. And you can't be sure they haven't misplaced their glasses.

Page 91 — Loci and Construction — Worked Examples

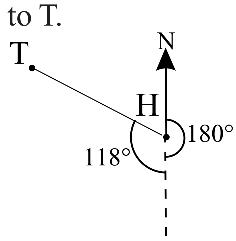
Q1 Shaded area = where public can go



[1 mark for loci 1 cm from pond. 1 mark for rounded corners of loci around pond. 1 mark for 1 cm circles around statues labelled A and B. 1 mark for correct region indicated.]

Page 92 — Bearings

Q1 Measure the clockwise angle from the north line to T.

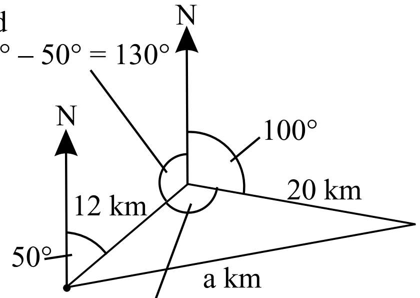


180° + 118° = 298° ***[1 mark]***

Worked Answers

Q2 First use the information given to draw a diagram. Then use angle rules to find other angles:

Using allied angles: 180° – 50° = 130°



Angles round a point: 360° – 100° – 130° = 130°

[1 mark for diagram showing both bearings given. 1 mark for using allied angles. 1 mark for using angles round a point.]

Cosine rule (see p99):

$a^2 = b^2 + c^2 - 2bc\cos A$

$= 12^2 + 20^2 - (2 \times 12 \times 20 \times \cos 130°)$ ***[1 mark]***

$= 852.53...$

So $a = \sqrt{852.53...} = 29.2$ km (1 d.p.) ***[1 mark]***

The question says "calculate", which is a whopping clue that you shouldn't measure. It also means that you don't need to draw a scale diagram — a sketch will suffice.

Pages 93-94 — Revision Questions

Q1 See page 71

Q2 **a)** First find missing angle in the triangle:
180 – 83 – 71 = 26°
Angles on a straight line add up to 180°, so
$x = 180 - 26 = 154°$

b) Using corresponding angles, $y = 112°$

c) Using alternate angles and the fact that this is an isosceles triangle, $z = 58°$

Q3 Exterior angle = 360 ÷ 6 = 60°
Sum of interior angles = (6 – 2) × 180° = 720°

Q4 Equilateral triangle:
lines of symmetry = 3
order of rotational symmetry = 3
Isosceles triangle:
lines of symmetry = 1
order of rotational symmetry = 1
Scalene triangle:
lines of symmetry = 0
order of rotational symmetry = 1

Q5 E.g. rhombus and parallelogram
An isosceles trapezium also has two pairs of equal angles.

Q6 See page 76-77

Q7 **a)** $x = 53°$ (angles in the same segment are equal)

b) $y = 90° - 21° = 69°$ (two radii form an isosceles triangle, and a tangent and a radius meet at 90°)

c) First find the angle at the centre using the rule that it is twice the angle at the circumference:
57° × 2 = 114°
Then use the rule that two radii form an isosceles triangle to find z:
$z = (180° - 114°) \div 2 = 33°$

Q8 Opposite angles in a cyclic quadrilateral add up to 180°, but 88° + 95° = 183° ≠ 180°, so the quadrilateral is not cyclic.

Q9 If two triangles are congruent, one of these conditions must hold: three sides are the same (SSS), two angles and a corresponding side match up (AAS), two sides and the angle between them match up (SAS), a right angle, the hypotenuse and one other side all match up (RHS).

Q10 E.g. Angles ACB and ACD are right angles (as it's a perpendicular bisector of a chord)
AB = AD (they're both radii)
CB = CD (as the chord is bisected)
So the condition RHS holds and the triangles are congruent.

Q11 Work out the scale factor from the pair of corresponding sides: 9 ÷ 3 = 3.
Now use the scale factor to work out required side length: $x = 7.5 \div 3 = 2.5$ cm

Q12 **a)** To get from A to B you move 7 units left and 5 units down. This is a translation by vector $\begin{pmatrix}-7\\-5\end{pmatrix}$ OR a rotation of 180° about point (1, 2).

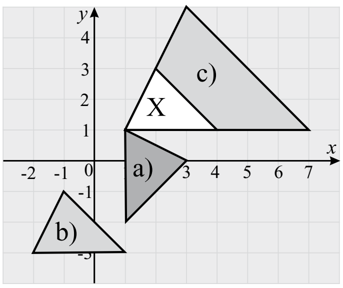
b) Shape C is an enlargement of scale factor $\frac{1}{3}$ and centre of enlargement (0, 0).

Worked Answers

Q13 **a)** See diagram below.

b) A translation of $\binom{-3}{-4}$ means move triangle X 3 units left and 4 units down. See diagram below.

c) The distance from the centre of enlargement to the bottom right-hand corner of X is 3 units. Multiply this distance by the scale factor 2 to get the distance of the corresponding corner of the image from the centre of enlargement: $3 \times 2 = 6$.
The top corner is 1 unit right and 2 units up from the centre of enlargement. So the image of this corner will be $1 \times 2 = 2$ units right and $2 \times 2 = 4$ units up from the centre of enlargement. The bottom left corner is on the centre of enlargement, so it stays put.



Q14 $A = ½(a + b) \times h_V$

Q15 Area of left-hand triangle $= 0.5 \times 6 \times 4 = 12$ cm^2
Area of right-hand triangle $= 0.5 \times 6 \times 5$
$= 15$ cm^2
Area of rectangle $= (16 - 4 - 5) \times 6 = 7 \times 6$
$= 42$ cm^2
Total area $= 12 + 15 + 42 = 69$ cm^2

Q16 Area = side length^2, so 56.25 = side length^2
side length $= \sqrt{56.25} = 7.5$ cm
Perimeter $= 4 \times$ side length $= 4 \times 7.5 = 30$ cm

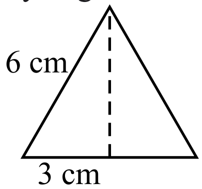
Q17 Circumference $= \pi \times D = 16\pi$ cm,
Area $= \pi r^2 = \pi \times (16 \div 2)^2 = 64\pi$ cm^2

Q18 Area of full circle $= \pi r^2 = \pi \times 10^2 = 100\pi$ cm^2
Area of sector $= \frac{45}{360} \times 100\pi = 39.27$ cm^2

Q19 Surface area of a sphere $= 4\pi r^2$
Surface area of a cylinder $= 2\pi rh + 2\pi r^2$
Surface area of a cone $= \pi rl + \pi r^2$

Q20 Area of curved part of cylinder
$= 2\pi r \times$ length $= 2\pi \times 3 \times 8 = 48\pi$ cm^2
Area of cylinder base $= \pi r^2 = \pi \times 3^2 = 9\pi$ cm^2
Area of hemisphere $= ½(4\pi r^2) = 2\pi \times 3^2$
$= 18\pi$ cm^2
Total surface area $= 48\pi + 9\pi + 18\pi$
$= 75\pi$ cm^2

Q21 A regular hexagon can be divided into 6 equilateral triangles of side length 6 cm. To find the area of one of these triangles, use Pythagoras' Theorem to find the height:



height $= \sqrt{6^2 - 3^2} = \sqrt{36 - 9} = \sqrt{27}$ cm
area of triangle $= 0.5 \times 6 \times \sqrt{27} = 3\sqrt{27}$ cm^2
area of hexagon $= 6 \times 3\sqrt{27} = 18\sqrt{27}$ cm^2
volume of prism $= 18\sqrt{27} \times 11 = 1028.838...$
$= 1030$ cm^3 (3 s.f.)

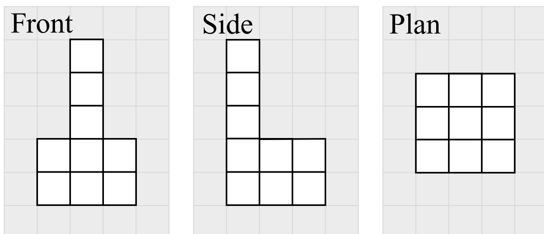
Q22 **a)** Volume of cylinder $= \pi r^2 h = \pi \times 2^2 \times 9$
$= 36\pi$ cm^3
Volume of cone $= \frac{1}{3}\pi r^2 h_v = \frac{1}{3}\pi \times 2^2 \times 4$
$= \frac{16}{3}\pi$ cm^3
Total volume $= 36\pi + \frac{16}{3}\pi = 129.852...$
$= 129.85$ cm^3 (2 d.p.)

b) 1.5 litres per minute = 1500 cm^3 per minute
= 25 cm^3 per second
Time taken to fill solid = 129.852... ÷ 25
= 5.194... seconds = 5.2 seconds (1 d.p.)

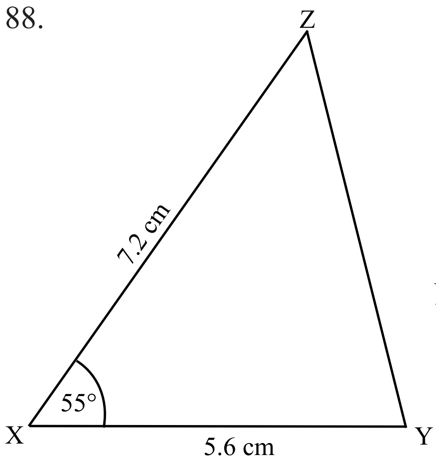
Q23 $5 \times 4^2 = 80$ cm^2

A scale factor of 4 makes the sides of a shape 4 times as long, but the area 16 times as big. It's one of those strange but true maths things.

Q24



Q25 Construct the triangle using a ruler and protractor, following the second method on page 88.

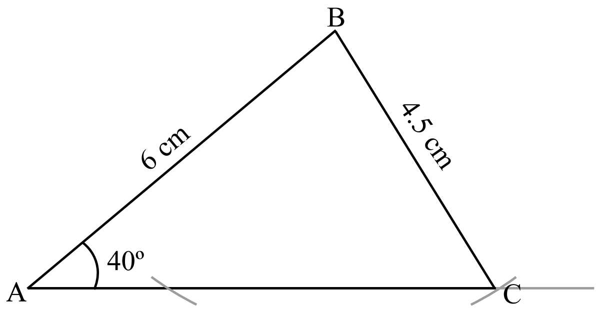


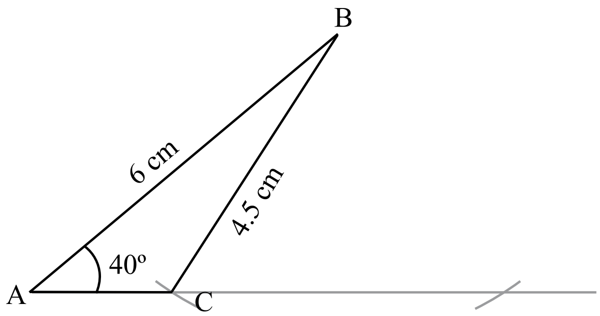
Not full size

A sharp pencil is the order of the day here. Take a spare into the exam so you don't have to faff around if your lead snaps.

Worked Answers

Q26 First draw a base line, mark point A and measure a 40° angle at A. Draw AB 6 cm long. Next set your compasses to 4.5 cm, put the point at B, and draw two arcs crossing your base line. The two intersection points are the possible locations of point C. Neither of these diagrams are full size.

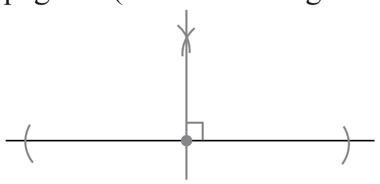


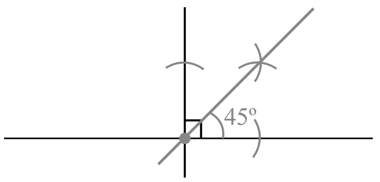


The fact that an SSA (or ASS) triangle doesn't give a single triangle is known as the "Donkey Theorem". Oh those mathematicians, they really crack me up.

Q27 A circle

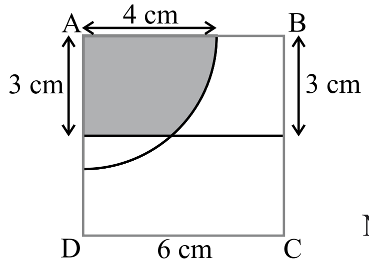
Q28 Construct a 90° angle using the method on page 90 (the first diagram below). Then bisect it to produce two 45° angles using the method on page 89 (the second diagram below).





Q29 See page 89

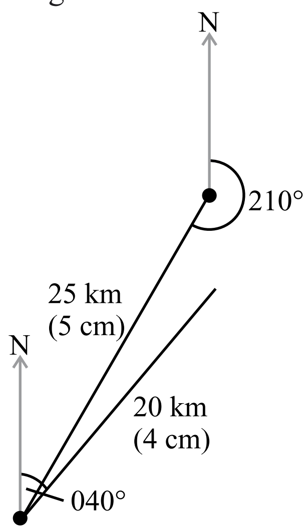
Q30 Measure halfway along sides AD and BC (3 cm), and use the marks to draw the locus of points equidistant from AB and CD. Then use compasses to draw the locus of points 4 cm from vertex A. Shade the required region.



Not full size

Q31 Put your pencil on the diagram at the point you're going FROM — point A. Draw a north line at this point. Measure the angle to the line AB clockwise from the north line — this is the bearing you want.

Q32 Using a scale of 1 cm = 5 km, 25 km is represented by a line 25 ÷ 5 = 5 cm long. 20 km is represented by a line 20 ÷ 5 = 4 cm long.



Section Six — Pythagoras and Trigonometry

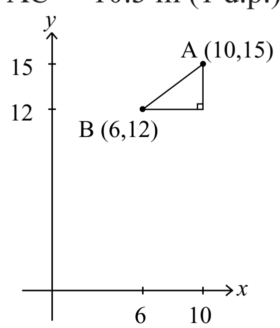
Page 95 — Pythagoras' Theorem

Q1 $5^2 + 9^2 = AC^2$ ***[1 mark]***

$AC = \sqrt{25 + 81} = \sqrt{106}$ ***[1 mark]***

AC = 10.3 m (1 d.p.) ***[1 mark]***

Q2



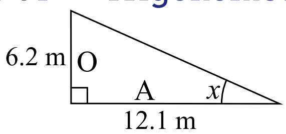
Worked Answers

Length of horizontal side = 10 – 6 = 4
Length of vertical side = 15 – 12 = 3 ***[1 mark]***
$AB^2 = 4^2 + 3^2$ ***[1 mark]***
$AB = \sqrt{16+9} = \sqrt{25}$ ***[1 mark]***
AB = 5 ***[1 mark]***

Q3 $a^2 + b^2 = (2\sqrt{10})^2$
$a^2 + b^2 = 40$ ***[1 mark]***
Now find two square numbers that add up to 40:
4 + 36 = 40 ***[1 mark]***
$2^2 + 6^2 = 40$
Possible lengths are 2 cm and 6 cm. ***[1 mark]***

Page 97 — Trigonometry — Examples

Q1



$T = \frac{O}{A}$

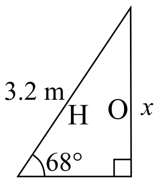
$\tan x = \frac{6.2}{12.1}$ ***[1 mark]***

$x = \tan^{-1}\left(\frac{6.2}{12.1}\right)$

$x = 27.1°$ (1 d.p.) ***[1 mark]***

Use the trig formula triangles for questions like this.

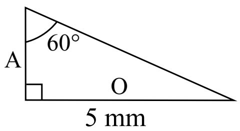
Q2



O = S × H
$x = \sin 68° \times 3.2$ ***[1 mark]***
$x = 2.97$ m (3 s.f.) ***[1 mark]***

Page 98 — Trigonometry — Common Values

Q1



$T = \frac{O}{A}$

$\tan 60° = \frac{5}{A}$ ***[1 mark]***

$A = \frac{5}{\tan 60°}$

$A = \frac{5}{\sqrt{3}}$ mm ***[1 mark]***

$\text{Area} = \frac{1}{2} \times 5 \times \frac{5}{\sqrt{3}}$ ***[1 mark]***

$= \frac{25}{2\sqrt{3}} = \frac{25\sqrt{3}}{6}$ mm² ***[1 mark]***

Page 99 — The Sine and Cosine Rules

Q1 $\text{Area} = \frac{1}{2} \times 9 \times 12 \times \sin 37°$ ***[1 mark]***
Area = 32.5 cm² (3 s.f.) ***[1 mark]***

Page 100 — The Sine and Cosine Rules

Q1 $\frac{AB}{\sin 38°} = \frac{24}{\sin 46°}$ ***[1 mark]***

$AB = \frac{24 \times \sin 38°}{\sin 46°}$ ***[1 mark]***

AB = 20.5 cm (3 s.f.) ***[1 mark]***

Q2 $13^2 = 9^2 + 15^2 - 2 \times 9 \times 15 \times \cos RPQ$

$\cos RPQ = \frac{81 + 225 - 169}{2 \times 9 \times 15}$ ***[1 mark]***

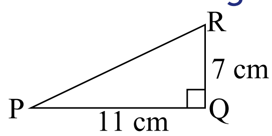
= 0.5074... ***[1 mark]***
$RPQ = \cos^{-1}(0.5074...) = 59.5°$ (3 s.f.) ***[1 mark]***

Page 101 — 3D Pythagoras

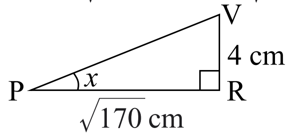
Q1 $AH^2 = 12^2 + 5^2 + 7^2$ ***[1 mark]***
$AH = \sqrt{144 + 25 + 49} = \sqrt{218}$ ***[1 mark]***
AH = 14.8 cm (3 s.f.) ***[1 mark]***

Page 102 — 3D Trigonometry

Q1



$PR^2 = 7^2 + 11^2$ ***[1 mark]***
$PR = \sqrt{49 + 121} = \sqrt{170}$ ***[1 mark]***



$\tan x = \frac{4}{\sqrt{170}}$ ***[1 mark]***

$x = \tan^{-1}\left(\frac{4}{\sqrt{170}}\right) = 17.1°$ (3 s.f.) ***[1 mark]***

Remember that though questions like this might look scary you can just use your standard Pythagoras and trig formulas.

Page 103 — Vectors

Q1 $\overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CB}$
$\overrightarrow{AB} = \mathbf{p} - 2\mathbf{q}$ ***[1 mark]***

$\overrightarrow{NA} = -\frac{1}{2}\overrightarrow{AB}$ ***[1 mark]***

$\overrightarrow{NA} = \mathbf{q} - \frac{1}{2}\mathbf{p}$ ***[1 mark]***

Worked Answers

Page 104 — Vectors

Q1 $\overrightarrow{AB} = \overrightarrow{AX} + \overrightarrow{XB}$

$\overrightarrow{AB} = \mathbf{a} - \mathbf{b}$ *[1 mark]*

$\overrightarrow{DC} = \overrightarrow{DX} + \overrightarrow{XC}$ *[1 mark]*

Using ratios: $\overrightarrow{DX} = \frac{3}{2}\overrightarrow{XB} = \frac{3}{2}(-\mathbf{b}) = -\frac{3}{2}\mathbf{b}$ *[1 mark]*

$\overrightarrow{XC} = \frac{3}{2}\overrightarrow{AX} = \frac{3}{2}\mathbf{a}$ *[1 mark]*

$\overrightarrow{DC} = -\frac{3}{2}\mathbf{b} + \frac{3}{2}\mathbf{a} = \frac{3}{2}(\mathbf{a} - \mathbf{b})$ *[1 mark]*

$\overrightarrow{DC}$ is a multiple of $\overrightarrow{AB}$ so these vectors are parallel and ABCD is a trapezium. *[1 mark]*

Page 105 — Revision Questions

Q1 $a^2 + b^2 = c^2$

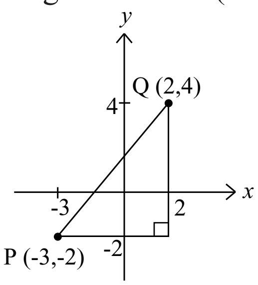
You use Pythagoras' theorem to find the missing side of a right-angled triangle.

Q2 $\text{length}^2 = 4^2 + 2.5^2$

$\text{length} = \sqrt{16 + 6.25} = \sqrt{22.25}$

length = 4.72 m (3 s.f.)

Q3



Length of horizontal side = 2 – (–3) = 5

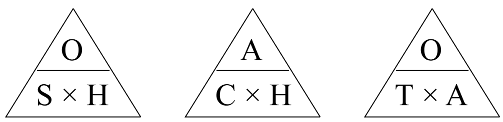
Length of vertical side = 4 – (–2) = 6

$PQ^2 = 5^2 + 6^2$

$PQ = \sqrt{25 + 36} = \sqrt{61}$

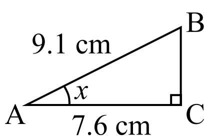
PQ = 7.8 (1 d.p.)

Q4



It's easy to write these down if you remember SOH CAH TOA.

Q5



$\cos x = \frac{\text{Adjacent}}{\text{Hypotenuse}}$

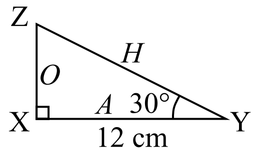
$\cos x = \frac{7.6}{9.1}$

$x = \cos^{-1}\left(\frac{7.6}{9.1}\right)$

$x = 33.4°$ (1 d.p.)

Q6 See page 98

Q7



$O = T \times A$

$= \tan 30° \times 12 = \frac{1}{\sqrt{3}} \times 12 = \frac{12}{\sqrt{3}}$

$= \frac{12\sqrt{3}}{3}$

$= 4\sqrt{3}$ cm

Don't forget to rationalise the denominator!

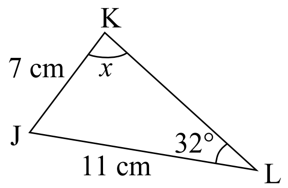
Q8 Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area $= \frac{1}{2}ab\sin C$

Q9
1. Two angles given plus any side — use the sine rule.
2. Two sides given plus an angle not enclosed by them — use the sine rule.
3. Two sides given plus the angle enclosed by them — use the cosine rule.
4. All three sides given but no angles — use the cosine rule.

Q10

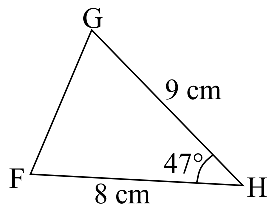


$\frac{7}{\sin 32°} = \frac{11}{\sin x}$

$\sin x = \frac{11\sin 32°}{7}$

$x = \sin^{-1}\left(\frac{11\sin 32°}{7}\right) = 56.4°$ (3 s.f.)

Q11

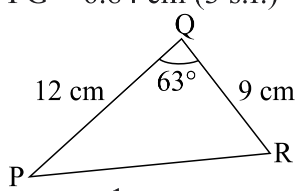


$FG^2 = 8^2 + 9^2 - 2 \times 8 \times 9 \times \cos 47°$

$FG = \sqrt{64 + 81 - 144 \times \cos 47°}$

FG = 6.84 cm (3 s.f.)

Q12



Area $= \frac{1}{2} \times 12 \times 9 \times \sin 63°$

Area = 48.1 cm^2 (3 s.f.)

Worked Answers

Q13 **a)** Angle $X = 180° - 112° - 46° = 22°$

$$\frac{5.25}{\sin 22°} = \frac{XZ}{\sin 112°}$$

$$\frac{5.25}{\sin 22°} \times \sin 112° = XZ$$

$XZ = 12.994\ldots$ m

$XY^2 = 12.994\ldots^2 + 8.5^2$
$\quad - 2 \times 12.994\ldots \times 8.5 \times \cos 26°$
$\quad = 42.55443\ldots$ m

$XY = 6.52337\ldots$ m

$XY = 6.52$ m (3 s.f.)

b) Area of $XYZ = \frac{1}{2} \times 12.994\ldots \times 8.5 \times \sin 26°$
$= 24.2092\ldots\ \text{m}^2$

Area of $WXZ = \frac{1}{2} \times 12.994\ldots \times 5.25 \times \sin 46°$
$= 24.5365\ldots\ \text{m}^2$

$24.2092\ldots + 24.5365\ldots = 48.7457\ldots\ \text{m}^2$
$= 48.7\ \text{m}^2$ (3 s.f.)

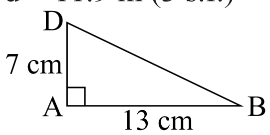
Q14 $a^2 + b^2 + c^2 = d^2$

Q15 $5^2 + 6^2 + 9^2 = d^2$

$d = \sqrt{25 + 36 + 81} = \sqrt{142}$

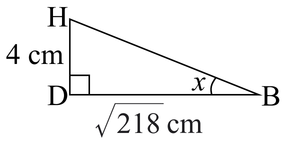
$d = 11.9$ m (3 s.f.)

Q16



$BD^2 = 7^2 + 13^2$

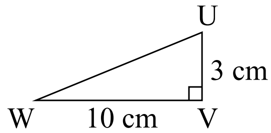
$PR = \sqrt{49 + 169} = \sqrt{218}$



$$\tan x = \frac{4}{\sqrt{218}}$$

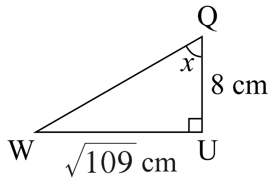
$$x = \tan^{-1}\left(\frac{4}{\sqrt{218}}\right) = 15.2° \text{ (3 s.f.)}$$

Q17



$WU^2 = 10^2 + 3^2$

$WU = \sqrt{100 + 9} = \sqrt{109}$



$$\tan x = \frac{\sqrt{109}}{8}$$

$$x = \tan^{-1}\left(\frac{\sqrt{109}}{8}\right) = 52.5° \text{ (1 d.p.)}$$

Q18 Multiplying by a scalar changes the size of a vector but not its direction.

Q19 **a)** $\begin{pmatrix}4\\-2\end{pmatrix} - \begin{pmatrix}7\\6\end{pmatrix} = \begin{pmatrix}4-7\\-2-6\end{pmatrix} = \begin{pmatrix}-3\\-8\end{pmatrix}$

b) $5 \times \begin{pmatrix}4\\-2\end{pmatrix} = \begin{pmatrix}20\\-10\end{pmatrix}$

c) $3 \times \begin{pmatrix}4\\-2\end{pmatrix} + \begin{pmatrix}7\\6\end{pmatrix} = \begin{pmatrix}12+7\\-6+6\end{pmatrix} = \begin{pmatrix}19\\0\end{pmatrix}$

d) $-4 \times \begin{pmatrix}4\\-2\end{pmatrix} - 2 \times \begin{pmatrix}7\\6\end{pmatrix} = \begin{pmatrix}-16\\8\end{pmatrix} - \begin{pmatrix}14\\12\end{pmatrix} = \begin{pmatrix}-30\\-4\end{pmatrix}$

Q20 **a)** $\overrightarrow{AX} = \frac{1}{3}\overrightarrow{XC}$

$\overrightarrow{AX} = \frac{1}{3}\mathbf{a}$

b) $\overrightarrow{DX} = \overrightarrow{DA} + \overrightarrow{AX}$

$\overrightarrow{DX} = \mathbf{a} - \mathbf{b} + \frac{1}{3}\mathbf{a}$

$\overrightarrow{DX} = \frac{4}{3}\mathbf{a} - \mathbf{b}$

$\overrightarrow{XB} = \overrightarrow{XA} + \overrightarrow{AB}$

$\overrightarrow{XB} = -\frac{1}{3}\mathbf{a} + 3\mathbf{a} - 2\mathbf{b}$

$\overrightarrow{XB} = \frac{8}{3}\mathbf{a} - 2\mathbf{b}$

c) $\overrightarrow{XB} = \frac{8}{3}\mathbf{a} - 2\mathbf{b} = 2\left(\frac{4}{3}\mathbf{a} - \mathbf{b}\right) = 2\overrightarrow{DX}$

$\overrightarrow{XB}$ is a scalar multiple of $\overrightarrow{DX}$ so DXB is a straight line.

Worked Answers

Section Seven — Probability and Statistics

Page 106 — Probability Basics

Q1 P(lands on 4)
= number of ways of getting a 4 ÷ number of possible outcomes = $\frac{3}{10}$ or 0.3
[2 marks available — 1 mark for using the correct formula and 1 mark for the correct probability.]

Q2 P(not red) = 1 – P(red)
= 1 – (1 – 3x) = 3x ***[1 mark]***

Page 107 — Counting Outcomes

Q1 **a)** List all the possible combinations:
HHH, HHT, HTH, THH, TTH, THT, HTT, TTT. ***[1 mark]***

b) There are 8 possible outcomes and 3 of them contain 2 heads so:
P(getting 2 heads) = $\frac{3}{8}$ or 0.375 ***[1 mark]***

Q2 P(no heads)
= number of ways to get no heads ÷ total number of possible outcomes
There are 2 outcomes each time you toss a coin and 10 coin tosses, so that's 2^{10} = 1024 possible outcomes. Only 1 outcome gives no heads so
P(no heads) = $\frac{1}{1024}$
[2 marks available — 1 mark for finding the total number of outcomes and 1 mark for the correct probability.]

Page 108 — Probability Experiments

Q1 **a)**

Score	Relative frequency
1	0.14
2	0.137
3	0.138
4	0.259
5	0.161
6	0.165

[2 marks for all six entries correct, or 1 mark for at least three entries correct.]

b) Yes, because the relative frequency for 4 is much higher than you'd expect from a fair dice (which is 1 ÷ 6 = 0.166...) ***[1 mark]***

Page 109 — Probability Experiments

Q1 **a)** Expected number = probability of 'Yes, No' × number of people
= 0.125 × 600 = 75
[2 marks available — 1 mark for using the correct formula, 1 mark for the correct answer]

b) Use the relative frequency of people taking A-level maths to estimate the probability:
Relative frequency = $\frac{30+9}{120}$ = $\frac{39}{120}$ = 0.325
0.325 × 600 = 195
[3 marks available — 1 mark for using the correct formula, 1 mark for the correct probability, 1 mark for the correct answer]

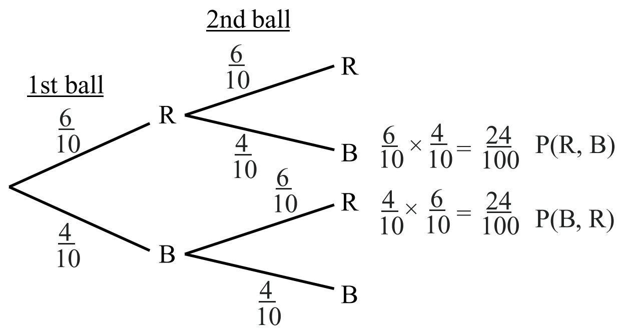
Page 110 — The AND / OR Rules

Q1 P(1st dice landing on an odd number) = $\frac{1}{2}$
P(2nd dice landing on an odd number) = $\frac{1}{2}$
P(he rolls two odd numbers) = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
[2 marks available — 1 mark for multiplying the probabilities, 1 mark for the correct answer.]

Q2 P(hoodie or jeans)
= P(hoodie) + P(jeans) – P(hoodie and jeans)
= 0.25 + 0.6 – 0.2 ***[1 mark]***
= 0.65 ***[1 mark]***

Page 111 — Tree Diagrams

Q1 Draw a tree diagram:

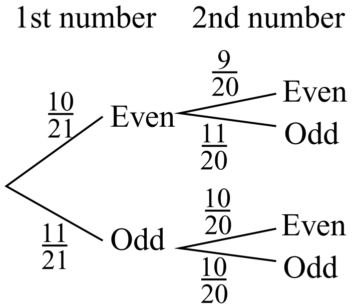


P(R, B) = $\frac{24}{100}$ and P(B, R) = $\frac{24}{100}$ ***[1 mark]***
So P(getting different colour balls)
= P(R, B or B, R) = $\frac{24}{100} + \frac{24}{100}$ ***[1 mark]***
= $\frac{48}{100} = \frac{12}{25}$ ***[1 mark]***

Worked Answers

Page 112 — Conditional Probability

Q1 There are 10 even numbers and 11 odd numbers.



a) P(at least 1 even) = 1 – P(both odd) ***[1 mark]***

$= 1 - \left(\frac{11}{21} \times \frac{10}{20}\right)$ ***[1 mark]***

$= 1 - \frac{110}{420} = \frac{310}{420} = \frac{31}{42}$ ***[1 mark]***

b) P(1 even and 1 odd)

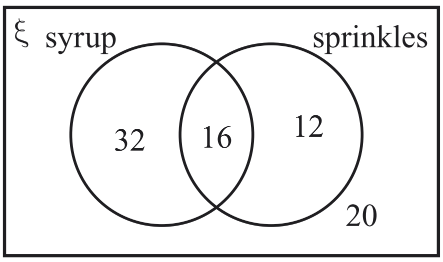
= P(even, odd) + P(odd, even) ***[1 mark]***

$= \left(\frac{10}{21} \times \frac{11}{20}\right) + \left(\frac{11}{21} \times \frac{10}{20}\right)$ ***[1 mark]***

$= \frac{110}{420} + \frac{110}{420} = \frac{220}{420} = \frac{11}{21}$ ***[1 mark]***

Page 113 — Sets and Venn Diagrams

Q1 48 – 16 = 32 only had syrup
28 – 16 = 12 only had sprinkles
80 – 32 – 16 – 12 = 20 had neither



P(no topping given no sprinkles)

$= \frac{\text{n(no topping)}}{\text{n(no sprinkles)}} = \frac{20}{52} = \frac{5}{13}$

[3 marks available — 1 mark for a completely correct Venn diagram, 1 mark for finding the number of customers having no sprinkles, 1 mark for the correct answer]

Alternatively you might use the conditional probability formula once you've completed the Venn diagram — if you got the correct answer you'd still get full marks.

Page 114 — Sampling and Data Collection

Q1 E.g. No, Tina can't use her results to draw conclusions about the whole population. The sample is biased because it excludes people who never use the train and most of the people included are likely to use the train regularly. The sample is also too small to represent the whole population.
[2 marks available — 1 mark for 'no', 1 mark for a correct comment about bias or sample size.]

Page 115 — Sampling and Data Collection

Q1 Discrete data ***[1 mark]***

E.g.

Cinema visits	Tally	Frequency
0-9		
10-19		
20-29		
30-39		
40-49		
50 or over		

[1 mark]

Page 116 — Mean, Median, Mode and Range

Q1 Mean = 79 ÷ 15 = 5.27 (3 s.f.) ***[1 mark]***.
There are 15 values, so the median is the (15 + 1) ÷ 2 = 8th value in the ordered list.
In order, the values are:
–14, –12, –5, –5, 0, 1, 3, 6, 7, 8, 10, 14, 18, 23, 25
So the median = 6 ***[1 mark]***.
The mode is the most common value.
So the mode = –5 ***[1 mark]***.
Range = greatest value – least value
= 25 – (–14) = 39 ***[1 mark]***.

Q2 Total of original 15 values = 79

$\frac{\text{New total}}{16} = 5.5$

So new total = 5.5 × 16 = 88 ***[1 mark]***
New value = 88 – 79 ***[1 mark]*** = 9 ***[1 mark]***

Page 117 — Frequency Tables — Finding Averages

Q1 a) There are 50 values, so the median is in position (50 + 1) ÷ 2 = 25.5, so it's halfway between the 25th and 26th values ***[1 mark]***.
Since these are both equal to 2, the median = 2 ***[1 mark]***.

Worked Answers

b) Add an extra column to the table, and an extra row for the totals.

No. of times sport played	Frequency	No. of times play sport × frequency
0	8	0
1	15	15
2	17	34
3	6	18
4	4	16
5 or more	0	0
Total	50	83

So the mean = 83 ÷ 50 = 1.66.

[3 marks available — 1 mark for working out the values in the extra column correctly, 1 mark for finding the correct total of 83, and 1 mark for a correct final answer.]

Page 118 — Grouped Frequency Tables

Q1 a) Extend the table:

Length (l cm)	$15.5 \leq l < 16.5$	$16.5 \leq l < 17.5$	$17.5 \leq l < 18.5$	$18.5 \leq l < 19.5$	Totals
Frequency (f)	12	18	23	8	61
Mid-interval value (x)	16	17	18	19	
fx	192	306	414	152	1064

So the mean $\approx \frac{1064}{61} = 17.4$ cm (3 s.f.)

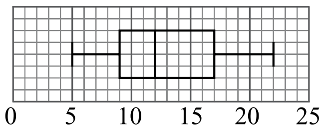
[4 marks available — 1 mark for using the correct mid-interval values, 1 mark for multiplying these by their corresponding frequencies, 1 mark for dividing the total of these results by the total of the frequencies, and 1 mark for a correct final answer.]

b) 12 out of 61 = 19.67...% of the lengths are below 16.5 cm ***[1 mark]***. ***[Either]*** Less than 20% of the lengths are below 16.5 cm, so Ana's statement is incorrect ***[1 mark]***.
[Or] Rounding to the nearest whole percent, 20% of the lengths are below 16.5 cm, so Ana's statement is correct ***[1 mark]***.
[2 marks available in total]

Sometimes there's more than one correct answer. Just make sure you draw a logical conclusion from the numbers you've got.

Page 119 — Box plots

Q1 You're told that the minimum value = 5, maximum value = 22 and IQR = 8.
50% of values are less than 12, so median = 12, and 75% of values are less than 17, so upper quartile = 17.
Work out the lower quartile:
lower quartile = upper quartile – IQR
= 17 – 8 = 9
So your box plot should look like this:

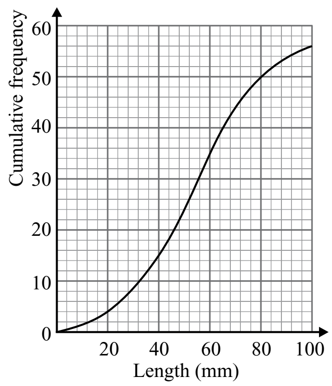


[3 marks available — 1 mark for the median in the correct place, 1 mark for both quartiles shown in the correct places, and 1 mark for both lowest and highest values shown in the correct places.]

Page 120 — Cumulative Frequency

Q1 a)

Length of fish (mm)	Frequency	Cumulative frequency
$0 < l \leq 20$	4	4
$20 < l \leq 40$	11	15
$40 < l \leq 60$	20	35
$60 < l \leq 80$	15	50
$80 < l \leq 100$	6	56



[3 marks available — 2 marks for all points correctly plotted (or 1 mark for at least 3 points correctly plotted), and 1 mark for drawing a smooth curve joining the points.]

b) From the graph, 32 fish are longer than 50 mm ***[1 mark for value in the range 30–34]***. (32 ÷ 56) × 100 = 57.1% (3 s.f.) ***[1 mark for answer in the range 53.6% – 60.7%]***.

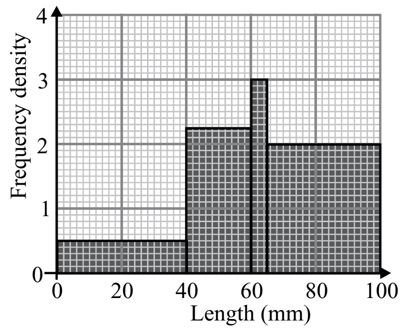
Worked Answers

Page 121 — Histograms and Frequency Density

Q1 a) Add an extra column to show frequency density.

Length (mm)	Frequency	Frequency density
$0 < x \le 40$	20	$20 \div 40 = 0.5$
$40 < x \le 60$	45	$45 \div 20 = 2.25$
$60 < x \le 65$	15	$15 \div 5 = 3$
$65 < x \le 100$	70	$70 \div 35 = 2$

Then draw the histogram.



[4 marks available — 1 mark for using the correct formula to calculate frequency density, 2 marks for all four frequency densities correct (or 1 mark for at least two correct), and 1 mark for a correctly drawn histogram.]

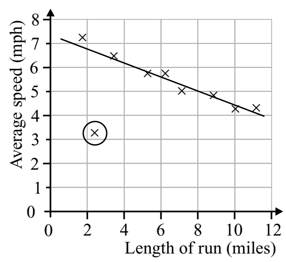
If you remember that it's the AREA of a bar that shows 'how many of something', then you shouldn't go far wrong.

b) Adding the frequencies for the first three classes: 20 + 45 + 15 = 80 slugs are shorter than 65 mm ***[1 mark]***. Number of slugs between 65 mm and 70 mm = area of 4th bar between 65 mm and 70 mm = 5 × 2 = 10 ***[1 mark]***.
So estimate of number shorter than 70 mm = 80 + 10 = 90 ***[1 mark]***.

Page 122 — Scatter Graphs

Q1 a) The length of run and the average speed have a strong negative correlation. The longer the run, the slower Sam's speed.
[1 mark]

b)



[1 mark for circling the correct point]

c) Draw a line of best fit (see graph above). Estimated speed is approximately 5 mph (±0.5 mph) ***[1 mark]***.

d) The estimate should be reliable because ***[either]*** 8 miles is within the range of the known data ***[1 mark]***, ***[or]*** the graph shows strong correlation ***[1 mark]***.

Page 123 — Other Graphs and Charts

Q1 There's a seasonal pattern that repeats itself every 4 points. The values are lowest in the first quarter and highest in the third quarter.
[1 mark]

Page 124 — Comparing Data Sets

Q1 Range of Science scores = 20 – 3 = 17
IQR of Science scores = 15 – 6 = 9
The range of Science scores is the same as the range of English scores, but the IQR for the Science scores is smaller, so Claudia is correct.
[2 marks available — 1 mark for comparing range and IQR, and 1 mark for a correct conclusion]

Page 125 — Comparing Data Sets

Q1 1) The data classes are unequal, so the columns shouldn't all be the same width ***[1 mark]***.
2) The horizontal axis isn't labelled ***[1 mark]***.
3) The frequency density scale isn't numbered ***[1 mark]***.

Page 126 — Revision Questions

Q1 There are 8 multiples of 6 between 1 and 50.
So P(number is a multiple of 6) $= \frac{8}{50} = \frac{4}{25}$

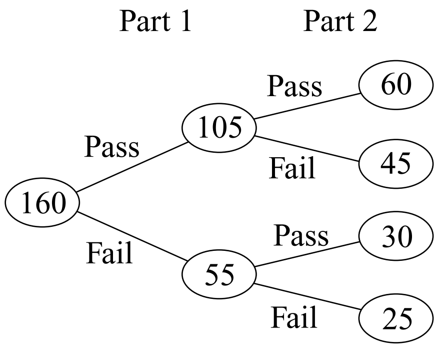
Q2 There are y outcomes for the first spin and y outcomes for the second spin. So, using the product rule, the total number of outcomes for the two spins $= y \times y = y^2$.
Number of ways to get two 1's = 1
P(getting two 1's) $= \frac{1}{y^2}$

You could also have used the AND rule here. The probability of getting a 1 each time is $\frac{1}{y}$, so $\frac{1}{y} \times \frac{1}{y} = \frac{1}{y^2}$.

Q3 Divide the frequency of each result by the number of times the experiment was tried.

Worked Answers

Q4 **a)** $160 - 105 = 55$ failed 1st part
$105 - 60 = 45$ passed 1st part but failed 2nd
$55 - 25 = 30$ failed 1st but passed 2nd



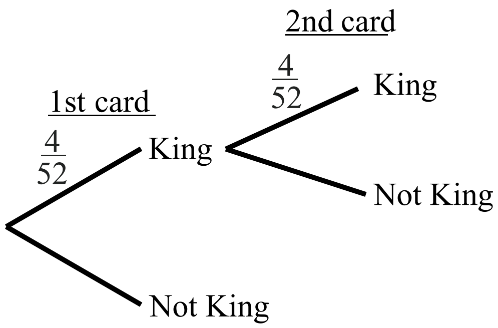
b) Relative frequency of:
pass, pass $= \frac{60}{160} = \frac{3}{8}$ or 0.375
pass, fail $= \frac{45}{160} = \frac{9}{32}$ or 0.28125
fail, pass $= \frac{30}{160} = \frac{3}{16}$ or 0.1875
fail, fail $= \frac{25}{160} = \frac{5}{32}$ or 0.15625

c) $300 \times 0.375 = 112.5$
$= 113$ (to the nearest whole number)

Q5 P(spinning a 6) $= \frac{1}{9}$,
P(spinning an even number) $= \frac{4}{9}$
P(a 6 then an even number) $= \frac{1}{9} \times \frac{4}{9} = \frac{4}{81}$

Q6 P(spinning factor of 20) $= \frac{6}{20}$
P(spinning an even number) $= \frac{10}{20}$
P(an even factor of 20) $= \frac{4}{20}$
P(factor of 20 or an even number)
$= \frac{6}{20} + \frac{10}{20} - \frac{4}{20} = \frac{12}{20} = \frac{3}{5}$

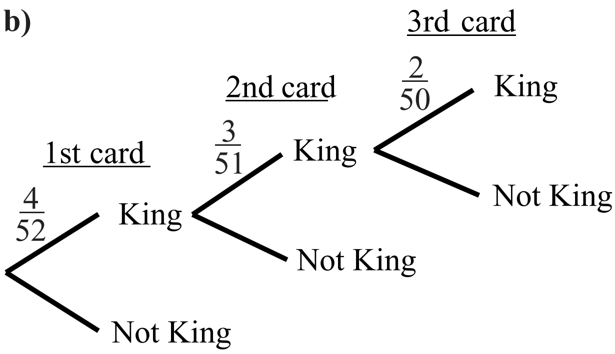
Q7 **a)**



So P(2 Kings) $= \frac{4}{52} \times \frac{4}{52} = \frac{16}{2704} = \frac{1}{169}$

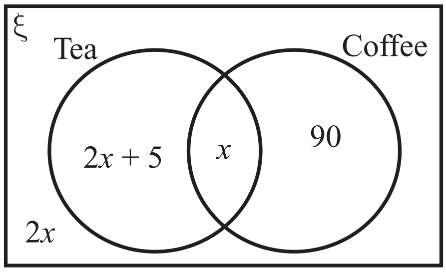
Remember to check with these 'picking' questions whether things are being replaced after each pick. It makes a big difference to the final answer.

b)



P(3 Kings)
$= \frac{4}{52} \times \frac{3}{51} \times \frac{2}{50} = \frac{24}{132600} = \frac{1}{5525}$

Q8 **a)**



b) $2x + (2x + 5) + x + 90 = 180$
$5x + 95 = 180$
$5x = 85$
$x = 17$

c) $(2x + 5) + x = 3x + 5$ people like tea, so
$(3 \times 17 + 5) = 56$ people like tea
P(liking tea) $= \frac{56}{180} = \frac{14}{45}$

Q9 A sample is part of a population. Samples need to be representative so that conclusions drawn from sample data can be applied to the whole population.

Q10 Qualitative data

Q11 Mode = 31
There are 11 values, so the median is the $(11 + 1) \div 2 = $ 6th value in the ordered list.
So median = 24.
Mean $= 242 \div 11 = 22$
Range $= 41 - 2 = 39$

Q12 **a)** Modal class is: $1.5 \le y < 1.6$.
b) There are 19 values, so the median is the $(19 + 1) \div 2 = $ 10th value in the ordered list. So the class containing the median is: $1.5 \le y < 1.6$

Worked Answers

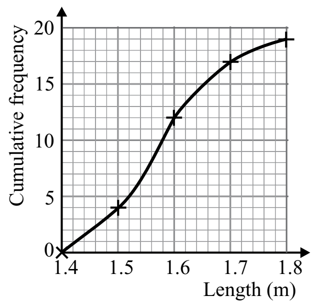
c) Add two extra columns to the table, and an extra row to show the totals you're going to need.

Length (y, in m)	Frequency (f)	mid-interval value (x)	fx
$1.4 \le y < 1.5$	4	1.45	5.8
$1.5 \le y < 1.6$	8	1.55	12.4
$1.6 \le y < 1.7$	5	1.65	8.25
$1.7 \le y < 1.8$	2	1.75	3.5
Total	19	-	29.95

So estimated mean = 29.95 ÷ 19
= 1.5763... = 1.58 m (to 2 d.p.)

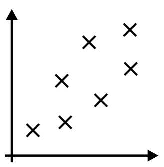
Q13

Length (y, in m)	Frequency	Cumulative frequency
$1.4 \le y < 1.5$	4	4
$1.5 \le y < 1.6$	8	12
$1.6 \le y < 1.7$	5	17
$1.7 \le y < 1.8$	2	19

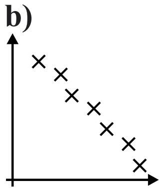


Q14 Calculate the bar's area, or use the formula:
frequency = frequency density × class width.

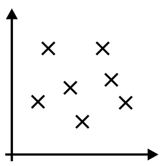
Q15 a)



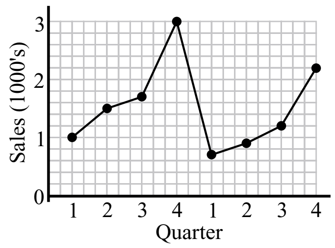
b)



c)



Q16 a)



b) The data shows a downward trend.

Q17 The median time in winter is lower than the median time in summer, so it generally took longer to get to work in the summer.
The range and the IQR for the summer are smaller than those for the winter, so there is less variation in journey times in the summer.
Try to think about what a low mean/median/mode would actually mean in the situation in the question (here, it's all about journey times). The same goes for your comment about the range and IQR.

Q18 Work out an estimate for the runner's mean time after the increase in training. Draw a table to help you estimate the total time she runs for:

Time for run (t seconds)	Frequency density	Frequency (f)	Midpoint (x)	fx
$130 \le t < 134$	1	4	132	528
$134 \le t < 138$	1.5	6	136	816
$138 \le t < 142$	1.75	7	140	980
$142 \le t < 146$	0.5	2	144	288
$146 \le t < 150$	0.25	1	148	148

$$\text{Mean time} = \frac{\text{Total time running}}{\text{Number of runs}} = \frac{528 + 816 + 980 + 288 + 148}{4 + 6 + 7 + 2 + 1} = \frac{2760}{20} = 138 \text{ seconds}$$

The runner's mean time after increasing her training hours has decreased from 147 seconds to 138 seconds, so this suggests that her running times have improved.